A Generalized Singularity Subtraction Method for Evaluating Layered Medium Green's Functions

Hongfeng Yang and Ali E. Yilmaz
Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, Texas, USA
yanghf14@utexas.edu, ayilmaz@mail.utexas.edu

***Abstract*—A novel spectral-domain singularity subtraction technique for accelerating the convergence of Sommerfeld integral tails is proposed for planar stratified media that include a perfect electrically conducting layer. Numerical results show that the extension avoids catastrophic cancellation in the spatial domain between the analytically computed and the numerically integrated terms, yields a rapidly decaying spectral tail, and enables accurate calculation of the Green's functions.**

Keywords—layered medium Green's function, singularity subtraction, perfect electric conductor

I. Introduction

Layered medium integral-equation solvers are used in a large variety of scattering and radiation problems [1],[2] and many of these involve highly conducting layers; e.g., for characterizing interconnects in electronic packages [3]. A rigorous technique for calculating layered medium Green's functions (LMGFs) is singularity subtraction [4]-[6]. In this approach, the Sommerfeld integral tail convergence is accelerated by first subtracting asymptotic forms of the spectral-domain LMGF kernels (as $k_\rho \to \infty$) from the integrand, numerically computing the asymptotically smoothed integral, and then adding the result of the numerical integration to the analytical transformation of the asymptotic forms in spatial domain. There are various shortcomings of this approach; e.g., for thin layers the asymptotic forms may not sufficiently increase the convergence of the tails until very large k_ρ [5] and naïve subtraction in finite-precision arithmetic can cause significant loss of accuracy [6].

Another important problem with the traditional singularity subtraction approach is the catastrophic cancellation in the spatial domain between analytically and numerically computed terms [7]; an issue that can arise when a perfect electric conductor (PEC) is present in the background medium. In the conventional singularity subtraction method, the direct term from the source to the observer is usually subtracted and evaluated analytically using Sommerfeld identity. The strong reflection term from the PEC interface vanishes, however, at the conventional singularity subtraction limit $k_\rho \to \infty$ in particular polarizations for non-magnetic media; thus it is completely numerically integrated. The few accurate significant digits computed from the numerical integral are catastrophically canceled by the analytically computed term in the spatial domain. This article presents a generalized subtraction method that can avoid this problem.

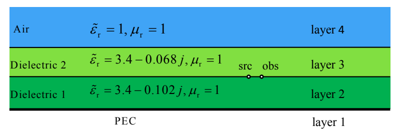

Fig 1. A PEC terminated non-magnetic stack-up based on [3]. Here, $\tilde{\varepsilon}_r$ is the complex relative permittivity; the thicknesses of dielectric 1 and dielectric 2 are 38.22 μm and 33.65 μm .

II. Formulation

A. Spatial-domain cancellation

Consider a non-magnetic layered medium terminated with a PEC denoted as layer 1 as in Fig. 1. For example, the vector potential component $K^{xx}\left(|\rho-\rho'|, z, z'\right) = K^{xx}_{\text{num}} + K^{xx}_{\text{ana}}$ is related to the following spectral-domain Green's function [1]:

$$\tilde{G}^h_{vi}(\mathbf{k}_\rho, z, z') = \frac{Z^h_{0,n}}{2}\left[e^{-jk^h_{z,n}|\Delta z|} + \frac{1}{D^h_n}\sum_{s=1}^{4} R^h_{n,s} e^{-jk^h_{z,n}\Delta z_{n,s}}\right] \quad (1)$$

Here, n is the layer number, $Z^h_{0,n}$ is the characteristic impedance of layer n for h polarization, $R^h_{n,1}$ is the transmission-line reflection coefficient looking to the right, i.e., upward direction, $R^p_{n,2}$ is the reflection coefficient looking to the left, $R^h_{n,3} = R^h_{n,4} = R^h_{n,1} R^h_{n,2}$. When the source and observer are in the third layer, $R^h_{n,2} e^{-jk^h_{z,n}\Delta z_{n,2}} \sim -e^{-2jk^h_{z,2}h_2} e^{-jk^h_{z,n}\Delta z_{n,2}}$ and the conventional singularity subtraction technique only subtracts the direct term because $R^h_{n,2} \to 0$ as $k_\rho \to \infty$; causing a cancellation problem between the numerical and analytical parts for large ρ . In effect, the fields radiated from the image of the source, which are numerically computed; cancel the analytically computed fields directly radiated from the source; the further away the observer is, the closer the two fields become and the more significant digits are lost.

B. Generalized subtraction

We observe that the cancellation problem can be avoided by subtracting a term $\beta e^{-jk^h_{z,n}\left(\Delta z_{n,s} + \chi h_{n-1}\right)}$, which can be interpreted as a perturbed image source, from the strong reflection term. The contribution of this term to the Green's function can be found analytically in the spatial domain via the Sommerfeld identity and be added back to the numerical term.

Consider the subtraction

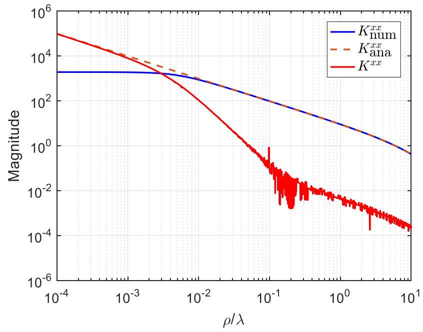

Fig. 2. The numerical part, analytical part, and the total spatial-domain K^{xx} using conventional singularity subtraction.

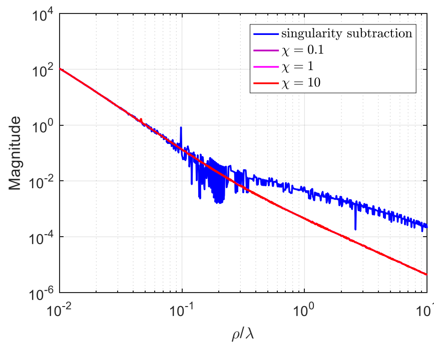

Fig. 3. The total spatial domain Green's function calculated using conventional singularity subtraction vs. the proposed method with different choices of χ , here, $\beta = -1$.

$$\frac{R^h_{n,s}}{D^h_n} e^{-jk^h_{z,n}\Delta z_{n,s}} - \beta e^{-jk^h_{z,n}\left(\Delta z_{n,s} + \chi h_{n-1}\right)} \quad (2)$$

When $\beta = 0$, the method reverts to traditional singularity subtraction for non-magnetic media; when $\beta = -1$, an exact image is subtracted. In general, β can depend on the parameters of the layer ($n-1$) in between the source/observer layer and the PEC. When the source and observer are both exactly at the interface, i.e., $\Delta z_{n,s} = 0$, a slight z-direction shift χh_{n-1} can be used; here, h_{n-1} is the thickness of the layer $n-1$ and χ is a tuning parameter that can be used to match the perturbed image source to the strong reflection term as well as to accelerate the decaying rate of the spectral integrand.

III. Numerical Results

To validate the method, we examine the four-layer non-magnetic stack of an electronic package (Fig. 1). The source and observer are both on the interface between the two dielectrics and ρ apart. The frequency is 20 GHz and the wavelength in the dielectric layers is $\lambda \approx 8.1$ mm. As shown in Fig. 2, using conventional singularity subtraction with a 10^{-3} numerical Sommerfeld integration threshold, the numerical part of K^{xx} approaches its analytical part when ρ gets large, consequently, there is a significant loss of digits in the total K^{xx} . The proposed method using the same threshold, as shown in Fig. 3, avoids the cancellation problem. Fig. 4 shows, when $\rho = \lambda$, the integrand of direct calculation, singularity subtraction, and generalized subtraction when $\chi = 0.1,\ 1, 10$. As can be seen, direct calculation (without any singularity subtraction) suffers from the slow decay of the integrand, whereas the generalized subtraction results in fast decaying integrands. In fact, a higher threshold can be used and fewer digits can be computed in the numerical Sommerfeld integral since the cancellation problem is already bypassed by subtracting the perturbed image source. This would result in a faster convergence of the numerical integral.

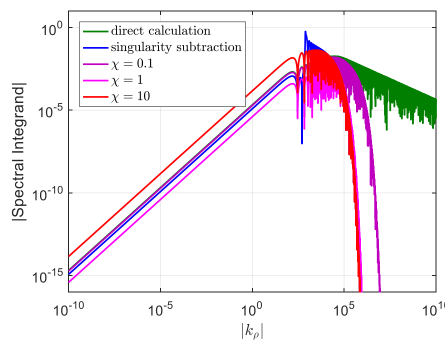

Fig. 4. Sampled spectral integrand of K^{xx} for direct calculation (no singularity subtraction), conventional singularity subtraction, and different choices of χ ($\beta = -1$).

References

[1] K. A. Michalski and J. R. Mosig, "Multilayered media Green's functions in integral equation formulations," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 3, pp. 508-519, Mar. 1997.

[2] K. Yang and A. E. Yilmaz, "A three-dimensional adaptive integral method for scattering from structures embeded in layered media," *IEEE Trans. Geosci. Remote Sensing*, vol. 50, no. 4, pp. 1130-1139, Apr. 2011.

[3] C. Liu, A. Menshov, V. Subramanian, K. Aygun, H. Braunisch, V. I. Okhmatovski, and A. E. Yilmaz, "Toward predictive modeling of full-size packages with layered-medium integral-equation methods", in *Proc. EPEPS*, Oct. 2016.

[4] E. Simsek, Q. H. Liu, and B. Wei, "Singularity subtraction for evaluation of Green's functions for multilayered media," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 1, pp. 216-225, Jan. 2006.

[5] M. A. Francavilla, D. R. Jackson, and D. R. Wilton, "Acceleration of the mixed-potential layered-medium Green's functions for problems with thin layers," in *Proc. ICEAA*, Oct. 2011.

[6] H. Yang and A. E. Yilmaz, "Accuracy preserving computation of asymptotically smoothed layered medium Green's functions in finite precision arithmetic," in *Proc. IEEE Antennas Propag. Soc. Int. Symp.*, July 2017.

[7] G. Bianconi, J. Pinto, and S. Chakraborty, "Efficient and robust dyadic Green's function evaluation algorithm for the analysis of IC packages and printed circuit boards," in *Proc. EPEPS,* Oct. 2016.