FFT-Accelerated Near-Field Scattering Evaluation

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***Abstract*—An FFT-based algorithm is presented for rapidly post-processing the integral-equation based solution of scattering problems to evaluate the fields at an arbitrary number of nearby points. The proposed algorithm uses a similar approach to the adaptive integral method (AIM) but contends with the fact that the fields are not Galerkin tested with basis functions but instead point tested. It reduces the computational costs compared to the brute-force method, especially when the number of observer points is large.**

I. Introduction

An integral part of any simulation is the calculation of the desired quantities of interest, which are often different from the solution quantities (that are output from the solver). While there have been advances in computational electromagnetics that have enabled the solution of large scale problems, post-processing algorithms are frequently overlooked and can even become more expensive than solving the original problem. Indeed, a simulator can be considered scalable, only if each step of it is scalable: setup, solve, *and* postprocessing [1].

Integral-equation methods are popular for many computational electromagnetics applications because their formulations requires the solution of a reduced set of unknowns when compared to the overall domain (i.e., only surfaces have unknowns for surface integral equation methods). However, multiphysics simulations may require a quantity of interest over the entire domain, e.g., to simulate local temperature increase from cell-phone radiation, the bioheat equation requires the absorbed power distribution everywhere. Additionally, for visualizing the fields (e.g., measuring magnetic fields induced by chip interrogation [2]), the quantities of interest also need to be calculated over the entire domain. A common quantity of interest for postprocessing is finding the fields at a set of N_{obs} observation points. As shown in the two examples, N_{obs} can be much greater than the original set of N unknowns.

For large N_{obs}, fast near-field evaluation is necessary. This abstract presents an FFT-based algorithm that is similar to the steps performed for a single matrix-vector multiplication in the adaptive integral method (AIM) [3].

II. Near-Field Scattering Evaluation Methods

In the method of moments (MoM), the unknown quantity that is solved for is approximated by a set of unknown coefficients ($\mathbf{I}$, $\mathbf{V}$) on N sub-domain basis functions ($\mathbf{f}_n$) on the discretized scattering object, i.e.,

$$\mathbf{J}(\mathbf{r}') \approx \sum_{n=1}^{N} \mathbf{I}[n]\mathbf{f}_n(\mathbf{r}') \quad \text{and} \quad \mathbf{M}(\mathbf{r}') \approx \sum_{n=1}^{N} \mathbf{V}[n]\mathbf{f}_n(\mathbf{r}'), \tag{1}$$

for electric and magnetic currents, respectively. With divergence-conforming basis functions (e.g., RWG [4], SWG [5], and volumetric rooftop [6] functions), the mixed-potential formulation is used for the relevant integral equations. After the solution, the now-known set of electric and magnetic current coefficients can be radiated to find the scattered fields

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) = -\eta_0\mathcal{L}_0\big(\mathbf{J}(\mathbf{r}'),\mathbf{r}\big) - \mathcal{K}_0\big(\mathbf{M}(\mathbf{r}'),\mathbf{r}\big) \tag{2}$$

$$\mathbf{H}^{\text{sca}}(\mathbf{r}) = \mathcal{K}_0\big(\mathbf{J}(\mathbf{r}'),\mathbf{r}\big) - \frac{\mathcal{L}_0\big(\mathbf{M}(\mathbf{r}'),\mathbf{r}\big)}{\eta_0}, \tag{3}$$

at observation position $\mathbf{r}$, where the $\mathcal{L}$ and $\mathcal{K}$ operators are

$$\mathcal{L}_0\big(\mathbf{v}(\mathbf{r}'),\mathbf{r}\big) = \gamma_0 \iiint_V \mathbf{v}(\mathbf{r}')g_0(d)\,\mathrm{d}V' - \frac{\boldsymbol{\nabla}}{\gamma_0}\iiint_V (\boldsymbol{\nabla}'\cdot\mathbf{v}(\mathbf{r}'))g_0(d)\,\mathrm{d}V' \tag{4}$$

$$\mathcal{K}_0\big(\mathbf{v}(\mathbf{r}'),\mathbf{r}\big) = \boldsymbol{\nabla}\times\iiint_V \mathbf{v}(\mathbf{r}')g_0(d)\,\mathrm{d}V', \tag{5}$$

and $d = |\mathbf{r}-\mathbf{r}'|$, $g_0(d) = e^{-\gamma_0 d}/4\pi d$ is the free-space Green's function, $\gamma_0 = j\omega\sqrt{\mu_0\varepsilon_0}$ is the propagation constant, and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free-space impedance. The total electric and magnetic fields are then found by adding the incident and scattered fields:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\text{inc}}(\mathbf{r})+\mathbf{E}^{\text{sca}}(\mathbf{r}) \text{ and } \mathbf{H}(\mathbf{r}) = \mathbf{H}^{\text{inc}}(\mathbf{r})+\mathbf{H}^{\text{sca}}(\mathbf{r}). \tag{6}$$

Note that for $\mathbf{r}$ inside or near a basis function, the singularity in the Green's function must be treated.

A. Brute-Force Method

To find the fields at a set of N_{obs} observation points, the brute-force method is to simply loop over the set of observers and find the fields radiated by the currents on the N basis functions. This naïve approach can be represented as a matrix-vector multiplication:

$$\begin{bmatrix} -\eta_0\mathbf{L}_u & -\mathbf{K}_u \\ \mathbf{K}_u & -\mathbf{L}_u/\eta_0 \end{bmatrix}\begin{bmatrix}\mathbf{I}\\ \mathbf{V}\end{bmatrix} = \begin{bmatrix}\mathbf{E}_u^{\text{sca}}\\ \mathbf{H}_u^{\text{sca}}\end{bmatrix} \quad \text{for } u\in\{x,y,z\}, \tag{7}$$

where $\mathbf{L}_u$ and $\mathbf{K}_u$ are $N_{\text{obs}}\times N$ matrices whose entries are

$$\mathbf{L}_u[m,n] = \big\langle \hat{\mathbf{u}}\delta(\mathbf{r}-\mathbf{r}_m)\,,\,\mathcal{L}_0\big(\mathbf{f}_n(\mathbf{r}'),\mathbf{r}\big)\big\rangle \tag{8}$$

$$\mathbf{K}_u[m,n] = \big\langle \hat{\mathbf{u}}\delta(\mathbf{r}-\mathbf{r}_m)\,,\,\mathcal{K}_0\big(\mathbf{f}_n(\mathbf{r}'),\mathbf{r}\big)\big\rangle, \tag{9}$$

and $\mathbf{E}_u^{\text{sca}}$ and $\mathbf{H}_u^{\text{sca}}$ are $N_{\text{obs}}\times 1$ vectors of the u-component of the fields at the N_{obs} observation points. While each entry of $\mathbf{L}_u$ and $\mathbf{K}_u$ must be calculated, they do not need to be explicitly stored if implemented inside of an observer loop. The inner product in (8) and (9) indicates non-Galerkin testing (as opposed to the corresponding MoM matrix entries that do Galerkin testing). The brute-force method requires $\mathcal{O}(NN_{\text{obs}})$ operations and negligible memory if the matrices are not explicitly stored.

B. Proposed Fast Method

First, a 3-D regular auxiliary grid of N_c points is created that encloses all the basis functions and observer points. Using this grid, the matrix blocks in (7) can be approximated as

$$\mathbf{L}_u \approx \mathbf{L}_u^{\text{FFT}} + \mathbf{L}_u^{\text{corr}} \quad \text{and} \quad \mathbf{K}_u \approx \mathbf{K}_u^{\text{FFT}} + \mathbf{K}_u^{\text{corr}}, \tag{10}$$

where

$$\begin{bmatrix} \mathbf{L}_x^{\text{FFT}} \\ \mathbf{L}_y^{\text{FFT}} \\ \mathbf{L}_z^{\text{FFT}} \end{bmatrix} = \bar{\bar{\mathbf{\Lambda}}}_{\text{obs}}^{\dagger} \begin{bmatrix} \mathbf{G} & \mathbf{0} & \mathbf{0} & \mathbf{G}^x \\ \mathbf{0} & \mathbf{G} & \mathbf{0} & \mathbf{G}^y \\ \mathbf{0} & \mathbf{0} & \mathbf{G} & \mathbf{G}^z \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}^x \\ \mathbf{\Lambda}^y \\ \mathbf{\Lambda}^z \\ \mathbf{\Lambda}^\nabla \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \mathbf{K}_x^{\text{FFT}} \\ \mathbf{K}_y^{\text{FFT}} \\ \mathbf{K}_z^{\text{FFT}} \end{bmatrix} = \bar{\bar{\mathbf{\Lambda}}}_{\text{obs}}^{\dagger} \begin{bmatrix} \mathbf{0} & -\mathbf{G}^z & \mathbf{G}^y \\ \mathbf{G}^z & \mathbf{0} & -\mathbf{G}^x \\ -\mathbf{G}^y & \mathbf{G}^x & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}^x \\ \mathbf{\Lambda}^y \\ \mathbf{\Lambda}^z \end{bmatrix} \tag{12}$$

$$\bar{\bar{\mathbf{\Lambda}}}_{\text{obs}}^{\dagger} = \begin{bmatrix} \mathbf{\Lambda}_{\text{obs}}^{\dagger} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_{\text{obs}}^{\dagger} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Lambda}_{\text{obs}}^{\dagger} \end{bmatrix}. \tag{13}$$

When multiplied with the current coefficient vectors, the proposed method does the following: (i) the currents are anterpolated onto the auxiliary grid via $\mathbf{\Lambda}^{\{x,y,z,\nabla\}}$; (ii) the sources on the auxiliary grid are radiated via $\{\mathbf{G}, \mathbf{G}^{\{x,y,z\}}\}$, resulting in fields on the auxiliary grid; (iii) the fields on the grid are interpolated to the observation points via $\mathbf{\Lambda}_{\text{obs}}^{\dagger}$; (iv) the fields radiated from nearby currents are corrected via $\mathbf{L}_u^{\text{corr}}$ and $\mathbf{K}_u^{\text{corr}}$.

The anterpolation matrices $\mathbf{\Lambda}^{\{x,y,z,\nabla\}}$ are sparse $N_\text{c} \times N$ matrices whose $\mathcal{O}(N)$ nonzero entries are found via moment matching the $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}, \nabla\} \cdot \mathbf{f}_n$ components of the n basis function to sources on the auxiliary grid [3], [7]. These anterpolation matrices are the same as those used in AIM [8], [9]. The propagation matrices, $\{\mathbf{G}, \mathbf{G}^{\{x,y,z\}} = \partial_{\{x,y,z\}}\mathbf{G}\}$, are dense $N_\text{c} \times N_\text{c}$ (three-level) block-Toeplitz matrices. They are also the same as the propagation matrices used in AIM with entries $\mathbf{G}[m,m] = 0$ (to avoid the singularity) and $\mathbf{G}[m,n] = g_0(|\mathbf{r}_m - \mathbf{r}_n|)$ for grid points $\mathbf{r}_m, \mathbf{r}_n$ [8], [9].

The interpolation matrix $\mathbf{\Lambda}_{\text{obs}}^{\dagger}$ is a sparse $N_{\text{obs}} \times N_\text{c}$ matrix whose nonzero entries are found via moment matching a source at each observer point to sources on the auxiliary grid [3], [7]. Unlike AIM, the proposed method is not Galerkin tested; thus, the $\mathbf{\Lambda}$ and $\mathbf{\Lambda}_{\text{obs}}$ matrices are different.

The correction matrices $\mathbf{L}_u^{\text{corr}}$ and $\mathbf{K}_u^{\text{corr}}$ are sparse matrices whose nonzero entries occur when observation point $\mathbf{r}_m$ is near the support of $\mathbf{f}_n(\mathbf{r}')$; their entries are given by

$$\mathbf{Z}_u^{\text{corr}}[m,n] = \mathbf{Z}_u[m,n] - \mathbf{Z}_u^{\text{FFT}}[m,n] \text{ for } \mathbf{Z} \in \{\mathbf{L}, \mathbf{K}\}. \tag{14}$$

The computational costs for the proposed fast method scale as follows: (i) $\mathcal{O}(N)$ operations to anterpolate the current and $\mathcal{O}(N + N_\text{c})$ bytes to store the currents; (ii) $\mathcal{O}(N_\text{c} \log N_\text{c})$ operations to propagate via 3-D FFTs the currents on the grid to fields on the grid and $\mathcal{O}(N_\text{c})$ bytes to store the field values on the grid; (iii) $\mathcal{O}(N_{\text{obs}})$ operations to interpolate the fields (when done on the fly inside of an observer loop, the memory cost is negligible); (iv) $\mathcal{O}(N_{\text{obs}})$ operations to fill and multiply the correction matrices (again, memory requirements are negligible when done on the fly). Thus, the proposed method requires a total of $\mathcal{O}(N + N_\text{c} \log N_\text{c} + N_{\text{obs}})$ operations and $\mathcal{O}(N + N_\text{c})$ bytes.

Note that, similar to AIM, N_c can be chosen such that it is proportional to N for volume integral equation methods and $N^{1.5}$ at worst for surface integral equation methods [8], [9].

III. Conclusion

Finding the scattered fields at many observation points is necessary for multiphysics applications (among others) and may easily become the dominant cost of the overall simulation if calculated naïvely as a postprocessing step following a fast solution algorithm. This abstract proposes a fast method that utilizes a regular auxiliary grid and FFTs similar to the matrix-vector multiplication in the AIM approach. At the conference, postprocessing results and performance data comparing brute-force and fast methods will be shown for large-scale bioelectromagnetic problems.

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