Direction Finding Algorithm for Noncircular Signals in the Presence of Unknown Mutual Coupling

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***Abstract*—Base on the fourth-order cumulants, a direction finding algorithm for noncircular sources under unknown mutual coupling is proposed. Utilizing the symmetric Toeplitz structure of the mutual coupling matrix and the noncircularity of the sources, a closed form solution for the DOA estimation is obtained by constructing a series of cumulant matrices. According to the simulation results, the proposed algorithm is effective in the presence of unknown mutual coupling. Moreover, it enjoys better estimation performance via utilizing the noncircularity of the sources.**

Keywords—DOA estimation, mutual coupling, noncircular signal, fourth-order cumulant.

I. INTRODUCTION

Noncircular signals, e.g., amplitude modulated (AM) and binary phase shift keying (BPSK) signals, are widely used in wireless communication systems [1]. By exploiting the noncircularity of these signals, one can improve the DOA estimation accuracy and increase the maximum number of detectable sources as well [2].

In [2], a MUSIC-like algorithm is developed by exploiting the complex conjugate counterpart of the received signals. In [3], a fourth-order cumulant-based DOA estimation algorithm is proposed for noncircular signals, which can detect more noncircular signals and has better performance than the MUSIC-like algorithm. However, these algorithms are generally based on the assumption of ideal array manifold which does not take the mutual coupling effect into account. In the literature [4,5], auto-calibration algorithms based on auxiliary sensors are developed. But they suffer from sever aperture loss since only the middle elements of the array are utilized. Therefore, it may not work under strong mutual coupling effects, since very few elements will be available under this scenario.

Aiming at the above-mentioned problems, we propose a cumulant-based direction finding algorithm, utilizing the symmetric Toeplitz structure of the coupling matrix and the noncircularity of the sources. Accordingly, the proposed algorithm enjoys the following advantages: 1) It makes use of the whole array elements and the noncirculartiy of the signals; 2) It is still applicable in strong mutual coupling condition; 3) Neither spectral search nor iterative processing is required, which makes it computationally efficient.

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II. SIGNAL MODEL

Suppose that there are K narrowband independent non-Gaussian signals sensed by a uniform linear array (ULA) composed of M omnidirectional elements. Considering the mutual coupling effect between the array elements, the observation vector can be modeled as

$$\mathbf{x}(t)=\sum_{k=1}^{K}\mathbf{Ca}(\theta_k)s_k(t)+\mathbf{n}(t)=\mathbf{CAs}(t)+\mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t)$, $\mathbf{a}(\theta)=[1,u(\theta),\cdots,u^{M-1}(\theta)]^T$, $\mathbf{A}$, $\mathbf{s}(t)$ and $\mathbf{n}(t)$ denote the received signal vector, the ideal steering vector, the ideal array manifold matrix, the source signal vector and the noise vector, respectively. $\mathbf{C}$ represents the mutual coupling coefficients matrix. As discussed in [4], the mutual coupling coefficient between two elements is inversely related to their distance. Therefore, $\mathbf{C}$ can be modeled as a banded symmetric Toeplitz matrix with P nonzero mutual coupling coefficients [4], which are arranged in vector $\mathbf{c}=[1,c_1,\cdots,c_{P-1}]^T=[1,\mathbf{c}_1^T]^T$.

Additionally, if the received signals are non-circular ones, it holds that $\mathbf{s}(t)=\mathbf{\Phi}^{1/2}\mathbf{s}_0(t)$, where $\mathbf{s}_0(t)$ is the zero-phase version of the sources, and the diagonal matrix $\mathbf{\Phi}^{1/2}$ contains the non-circularity phase shifts of the signals. Therefore, the array output for noncircular signals in the presence of mutual coupling can be given by

$$\mathbf{x}(t)=\mathbf{CA\Phi}^{1/2}\mathbf{s}_0(t)+\mathbf{n}(t) \tag{2}$$

III. PROPOSED ALGORITHM

According to the signal model introduced above, the steering vector of the array can be parameterized as follows [6]

$$\bar{\mathbf{a}}(\theta)=\mathbf{Ca}(\theta)=\tau(\theta)\mathbf{g}(\theta)\circ\mathbf{a}(\theta) \tag{3}$$

herein, $\mathbf{g}(\theta)=\text{diag}\{\nu_1,\cdots,\nu_{P-1},1,\cdots,1,\eta_1,\cdots,\eta_{P-1}\}$ is an $M\times M$ diagonal matrix. The definition of τ, ν_P and η_P can be referred to [6], and a total of $\bar{M}=M-2P+2$ ones are located between ν_{P-1} and η_1. Based on this property, it holds that $\bar{a}_{P+i}(\theta)/\bar{a}_P(\theta)$ $=a_{P+i}(\theta)/a_P(\theta)=u^i(\theta)$, $i=0,1,\cdots,\bar{M}-1$, where $\bar{a}_m(\theta)$ and $a_m(\theta)$ are the mth elements of $\mathbf{a}(\theta)$ and $\bar{\mathbf{a}}(\theta)$, respectively. Therefore, we can construct a $M\times M$ fourth-order cumulant matrix $\mathbf{C}_{1i}$, with its (m,n)th element being

$$\mathbf{C}_{1i}(m,n) = \text{cum}\left(x_{P+i}(t), x_P^*(t), x_m(t), x_n^*(t)\right) \tag{4}$$

where $m,n = 1,2,\cdots,M$ and $i \in [0, \bar{M}-1]$. Because the sensor noises are zero mean Gaussian processes, the above expression can be further written as:

$$\mathbf{C}_{1i} = \sum_{k=1}^{K} u^i(\theta_k)\alpha(\theta_k)\gamma_k \bar{\mathbf{a}}(\theta_k)\bar{\mathbf{a}}^H(\theta_k) = \bar{\mathbf{A}}\boldsymbol{\Omega}^i \mathbf{C}_{4s}\bar{\mathbf{A}}^H \tag{5}$$

where $\alpha(\theta) = \bar{a}_P(\theta)\bar{a}_P^*(\theta)$, $\gamma_k = \text{cum}(s_k(t), s_k^*(t), s_k(t), s_k^*(t))$ and $\boldsymbol{\Omega} = \text{diag}\{u(\theta_1),\cdots,u(\theta_K)\}$ is a $K\times K$ diagonal matrix, which contains the DOA information of the K sources. $\mathbf{C}_{4s} = \text{diag}\{\gamma_1\alpha(\theta_1),\cdots,\gamma_K\alpha(\theta_K)\}$ is a $K\times K$ diagonal matrix, and it is easy to verify that $\mathbf{C}_{4s} = \mathbf{C}_{4s}^*$.

Similarly, we can construct other three cumulant matrices $\mathbf{C}_{2i}$, $\mathbf{C}_{3i}$, $\mathbf{C}_{4i}$, with their (m,n)th elements being $\mathbf{C}_{2i}(m,n) = \text{cum}(x_{P+i}, x_P^*, x_m, x_n)$, $\mathbf{C}_{3i}(m,n) = \text{cum}(x_P, x_{P+i}^*, x_m, x_n)$ and $\mathbf{C}_{4i}(m,n) = \text{cum}(x_{P+i}, x_P^*, x_m^*, x_n)$. Following the above derivation, $\mathbf{C}_{2i}$, $\mathbf{C}_{3i}$ and $\mathbf{C}_{4i}$ can be expressed in matrix form: $\mathbf{C}_{2i} = \bar{\mathbf{A}}\boldsymbol{\Omega}^i\mathbf{C}_{4s}\boldsymbol{\Phi}\bar{\mathbf{A}}^T$, $\mathbf{C}_{3i} = \bar{\mathbf{A}}\boldsymbol{\Omega}^{-i}\mathbf{C}_{4s}\boldsymbol{\Phi}\bar{\mathbf{A}}^T$, and $\mathbf{C}_{4i} = \bar{\mathbf{A}}^*\boldsymbol{\Omega}^i\mathbf{C}_{4s}\bar{\mathbf{A}}^T$.

It is obvious that the non-circularity phase information is embedded in $\mathbf{C}_{2i}$ and $\mathbf{C}_{3i}$. Therefore, we can construct the following augmented matrix to improve the DOA estimation.

$$\mathbf{C}_i = \begin{bmatrix} \mathbf{C}_{1i} & \mathbf{C}_{2i} \\ \mathbf{C}_{3i}^* & \mathbf{C}_{4i} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{A}} \\ \bar{\mathbf{A}}^*\boldsymbol{\Phi}^* \end{bmatrix} \boldsymbol{\Omega}^i \mathbf{C}_{4s} \begin{bmatrix} \bar{\mathbf{A}} \\ \bar{\mathbf{A}}^*\boldsymbol{\Phi}^* \end{bmatrix}^H = \mathbf{B}\boldsymbol{\Omega}^i\mathbf{C}_{4s}\mathbf{B}^H \tag{6}$$

where $\mathbf{B}$ is a $2M\times K$ augmented manifold matrix. It is worthwhile to notice that $\mathbf{B}$ extends the virtual aperture size of the array, and this property could be utilized to improve the estimation accuracy and resolve more sources.

For different integer values of i ($i \in [0, \bar{M}-1]$), a total of $\bar{M}$ cumulant matrices $\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_{\bar{M}}$ can be constructed. By concatenating them together, a tall matrix $\bar{\mathbf{C}}$ is built as follows

$$\bar{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_0 \\ \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{\bar{M}-1} \end{bmatrix} = \begin{bmatrix} \mathbf{B} \\ \mathbf{B}\boldsymbol{\Omega} \\ \vdots \\ \mathbf{B}\boldsymbol{\Omega}^{\bar{M}-1} \end{bmatrix} \mathbf{C}_{4s}\mathbf{B}^H \tag{7}$$

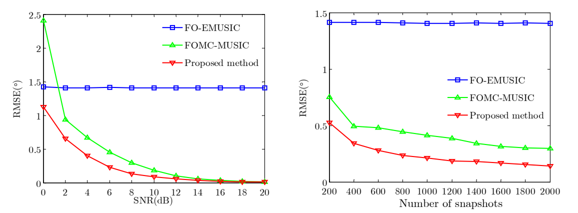
By taking the SVD of $\bar{\mathbf{C}}$, we have $\bar{\mathbf{C}} = \mathbf{U}_s\mathbf{D}\mathbf{V}$, where $\mathbf{D}$ is the diagonal matrix containing K nonzero singular values, $\mathbf{U}_s$ is composed of the left singular vectors, and $\mathbf{V}$ is composed of the right singular vectors. Note that $\mathbf{U}_s$ can be partitioned into two overlapping submatrices $\mathbf{U}_1$ and $\mathbf{U}_2$, which are composed of the first $2M(\bar{M}-1)$ and last $2M(\bar{M}-1)$ rows of $\mathbf{U}_s$, respectively. Therefore, based on the subspace rotational invariance property, it can be readily derived that $\boldsymbol{\Omega} = \mathbf{T}^{-1}(\mathbf{U}_1^H\mathbf{U}_1)^{-1}\mathbf{U}_1^H\mathbf{U}_2\mathbf{T}$. Then, by taking the EVD of $(\mathbf{U}_1^H\mathbf{U}_1)^{-1}\mathbf{U}_1^H\mathbf{U}_2$ to obtain its eigenvalues $\omega_1, \omega_2, \cdots, \omega_K$, the DOAs can be determined as follows:

$$\theta_k = \sin^{-1}\left(\frac{\lambda \arg(\omega_k)}{2\pi d}\right), \quad k = 1,2,\cdots,K \tag{8}$$

Since the DOA estimates are obtained in closed form, the algorithm is computationally more efficient than those spectral search based algorithms.

IV. Simulation Results

In this section, several numerical simulations are conducted to validate the performance of the proposed algorithm relative to FO-EMUSIC [3] and FOMC-MUSIC [5]. In the following simulations, we consider a ULA composed of 8 elements with $d=\lambda/2$. The impinging sources are equi-power, independent BPSK signals. The performance is measured by the root mean-square error (RMSE) of 500 independent Monte Carlo runs. The scenario of strong mutual coupling is investigated, with P=4 and the nonzero MCCs being c_1= 0.1545+j0.4755, c_2=-0.1618+j0.1176, c_3=-0.0211+ j0.0651. Note that, under this scenario, only 2 elements can be utilized in FOMC-MUSIC. However, the proposed algorithm utilized the whole array and the noncircularity of the signals. The corresponding RMSEs versus SNR and the number of snapshots are shown in Figs. 1. For FOMC-MUSIC and the proposed algorithm, the RMSEs decrease monotonically with the increase of SNR and the number of snapshots. For FO-EMUSIC, however, increasing the SNR and the number of snapshots is no longer helpful. As is expected, both FO-EMUSIC and FOMC-MUSIC are outperformed by our proposed method.



(a). RMSEs versus SNR (b). RMSEs versus number of snapshots

Fig. 1. RMSEs of DOA estimation versus SNR and number of snapshots.

V. Conclusion

In this paper, a cumulants based direction finding algorithm is proposed for noncircular signals under unknown mutual coupling effect. Compared with some existing methods, the new method is effective in the presence of unknown mutual coupling and utilizes the whole array to determine the unknown DOAs. Moreover, our method is efficient in that it successfully circumvents any multidimensional spectral search. Furthermore, compared with the conventional FOC-based algorithms, the proposed algorithm enjoys better estimation performance via utilizing the noncircularity of the sources.

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