A Novel Algorithm Based MVDR Beamformer

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***Abstract*— MVDR beamformers using data transformed by applying R-1D-MRT algorithm are discussed here. These beamformers exhibit reduction in computation time and computational complexity and improved performance in AWGN channels. They pass signals from the desired direction with unity gain and rejects signals from undesired directions.**

Keywords—beamforming; computational complexity; computation time; MVDR; R-1D-MRT

I. INTRODUCTION

The Minimum Variance Distortionless Response (MVDR) beamformer is constrained to pass signals from the desired direction with unity gain and reject signals from undesired directions. In the R-1D-MRT MVDR beamformer the input data is transformed by applying Reduced One Dimensional Mapped Real Transform (R-1D-MRT) algorithm and then adaptive MVDR beamforming is performed using this transformed data. This paper discusses the reduction in computational complexity and computation time and performance improvement in AWGN channels obtained with the R-1D-MRT MVDR beamformer.

II. R-1D-MRT ALGORITHM

The 1D Mapped Real Transform (1D-MRT) exploits the periodicity and symmetry of exponential terms in the DFT and groups related data for analyzing one dimensional signals in the frequency domain using only addition operations. For obtaining 1D-MRT, consider a 1-D sequence x_n, $0 \le n \le N-1$ with Y_k, $0 \le k \le N-1$ as its DFT given by

$$Y_K = \sum_{n=0}^{N-1} x_n\, e^{\frac{-j2\pi nk}{N}} = \sum_{n=0}^{N-1} x_n\, W_N^{((nK))_N} \quad (1)$$

where k is the frequency index. The exponent $((nk))_N$ can have a value p, where p is the phase index with $0 \le p \le N-1$. For a given value of k, by grouping the data that share the same value p for the exponent $((nk))_N$, and also using the relation $W_N^{p+\frac{N}{2}} = W_N^p$, Y_k can be expressed as

$$Y_K = \sum_{n=0}^{N-1} Y_k^p\, W_N^p \quad (2)$$

where Y_k^P is the 1D–MRT of x_n, $0 \le n \le N-1$, defined as

$$Y_k^{(p)} = \sum_{\forall n \Rightarrow ((nk))_N = p} x_n \sum_{\forall n \Rightarrow ((nk))_N = p+M} x_n \quad (3)$$

with M=N/2. The 1-D MRT maps a one dimensional array of length N into M one dimensional arrays, each of length N, resulting in a matrix of M x N elements using only additions for computing the MN coefficients[1][2] and is valid for all even values of N.

In the R-1D-MRT algorithm proposed for beamforming, only elements of the 1D-MRT corresponding to frequency index k=1 is used for beamforming, subject to the signal being sampled at a frequencies specific to the size of the input data vector [3]. The Forward R-1D-MRTis expressed as:

$$\mathbf{Y}_{R-1D-MRT} = \mathbf{X}_{n_{[1x(1:\frac{N}{2})]}} - \mathbf{X}_{n_{[1x(\frac{N}{2}+1:N)]}} \quad (4)$$

The original data can be reconstructed from the R-1D-MRT beam former output **Y** using the relation:

$$\mathbf{X} = [\tfrac{\mathbf{Y}}{2}; -\tfrac{Y}{2}] \quad (5)$$

The optimum sampling frequencies for few data sizes is given in Table 1

Table 1: Optimum Sampling frequencies

N (Data size)	Sampling frequency (fs)
1024	4.0470f
512	4.0960f
256	4.6550f
128	4.1290f
64	4.9300f

The size of the data for beamforming reduces to half the original size by application of R-1D-MRT algorithm.

III. MVDR BEAMFORMER

In the MVDR beamformer the weights are chosen to maintain a distortionless response with unity gain for the signals in the desired direction θ_d while minimizing the output power of the interfering signals and noise [4]. The output power of the MVDR beamformer is expressed as:

$$P = [E(\mathbf{Y}^2)] = E[\mathbf{W}^H\mathbf{X}\mathbf{X}^H\mathbf{W}] = \mathbf{W}^H E[\mathbf{X}\mathbf{X}^H]\mathbf{W} = \mathbf{W}^H C \quad (6)$$

where **X** is the array input signal, **W** is the weight vector and H is Hermitian transpose, $C = E[XX^H]$ is the correlation matrix of the received signal and E(.) is the expectation operator. The MVDR adaptive algorithm is represented as:

$$\min_W\{\mathbf{W}^H C\mathbf{W}\} \text{ subject to } \mathbf{W}^H\mathbf{a}(\theta_d) = 1 \quad (7)$$

The minimization of the total output noise, while maintaining the output signal constant, is equivalent to maximizing the output SINR in the desired direction. The optimized weight vector of the MVDR beamformer in terms of the steering vector is expressed as

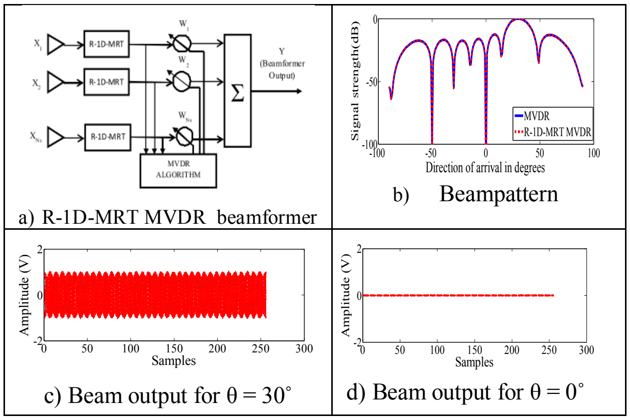
$$W_{MVDR} = [w_1 \quad w_2 \quad \cdots \quad \cdots \quad w_{N_S}]^T \quad (8)$$

$$W_{MVDR} = \frac{C^{-1}a(\theta_d)}{a^H(\theta_d)C^{-1}a(\theta_d)} \quad (9)$$

In the R-1D-MRT MVDR beamformer, the signals received at the array are transformed using the R-1D-MRT algorithm and then multiplied with the MVDR weights and then summed.

1V. SIMULATION RESULTS

The simulation results obtained with R-1D-MRT MVDR beamformer for an array of Ns = 8 sensors and data size N = 256 with desired signal at 30º and interfering signals at 0º and -50º is shown in Fig 1.



a) R-1D-MRT MVDR beamformer b) Beampattern
c) Beam output for θ = 30˚ d) Beam output for θ = 0˚

Fig 1. Block diagram,beam pattern and simulation results of R-1D-MRT MVDR beamformer

The beampattern in Fig 1(b) exhibits a peak in the desired direction and nulls in the interfering directions. The beam output is constrained to 1 for desired direction as seen in Fig 1(c).The reduction in the size of the data by application of R-1D-MRT algorithm results in reduction in computational complexity and computation time as shown in Tables 2 and 3.

Table 2: Comparison of computational complexity

N (Data size)	$N_s = 8$			
	Number of real multiplications		Number of real additions	
	MVDR BF	R-1D-MRT MVDR BF	MVDR BF	R-1D-MRT MVDR BF
8192	2359008	1179360	2358880	1179344
4096	1179360	589536	1179232	589520
2048	589536	294624	589408	294608
1024	294624	147168	294496	147152
512	147168	73440	147040	73424
256	73440	36576	73312	36560
128	36576	18144	36448	18128
64	18144	8928	18016	8912
32	8928	4320	8800	4304
16	4320	2016	4192	2000

It can be observed from Table 2 that the R-1D-MRT MVDR beamformer requires approximately half the number of real multiplications and additions compared to conventional MVDR beamformer.

Table 3: Comparison of computation time in seconds

N (data size)	Ns = 8		Ns = 50		Ns = 100	
	MVDR BF	R-1D-MRT MVDR BF	MVDR BF	R-1D-MRT MVDR BF	MVDR BF	R-1D-MRT MVDR BF
8192	0.0099	0.0051	0.0352	0.0196	0.0927	0.0482
4096	0.0051	0.0029	0.0183	0.0120	0.0494	0.0258
2048	0.0034	0.0021	0.0098	0.0074	0.0244	0.0157
1024	0.0022	0.0016	0.0059	0.0043	0.0149	0.0105
512	0.0016	0.0011	0.0039	0.0032	0.0084	0.0079
256	0.0012	0.0008	0.0028	0.0025	0.0059	0.0058
128	0.0009	0.0008	0.0025	0.0021	0.0049	0.0046
64	0.0009	0.0008	0.0019	0.0018	0.0042	0.0041

Table 3 shows a significant reduction in computation time for the R-1D-MRT MVDR beamformer especially with increase in the data size and number of sensors. The computation was performed using computer with Intel Core i5 CPU working at 2.20 GHz and RAM of 4GB. Scatterplots showing the performance of R-1D-MRT MVDR beamformer in AWGN channel is presented in Fig 2.

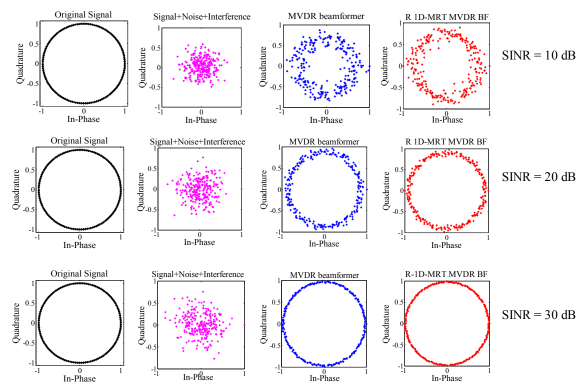


Fig 2:Scatterplots for AWGN channel

The scatterplots in Fig 2 show less scattering for R-1D-MRT MVDR beamformer compared to conventional beamformer.

The above results indicate that the R-1D-MRT MVDR beamformer shows a reduced computational complexity and computation time and improved performance in AWGN channels while satisfying the constraint of unity gain in the desired direction and rejection of interfering signals.

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