Coupled Loop Resonator Hilbert Transformer

Xiaoyi Wang, Lianfeng Zou, Zoé-Lise Deck-Léger, Christophe Caloz
Department of Electrical Engineering, Polytechnique Montréal
Montréal, Québec, H3T 1J4, Canada
xiaoyi.wang@polymtl.ca

José Azaña
Institut National de la Recherche Scientifique – Énergie Matériaux et Télécommunications (INRS-EMT)
Varennes, Québec J3X 1S2, Canada

***Abstract*—We characterize the coupled loop resonator Hilbert transformer (HT) and subsequently propose an ideal implementation if in the form of a cascade of two 3 dB units. This HT, that will be experimentally characterized and demonstrated in applications at the conference, will play an important role as a basic operator for Radio Analog Processing (RAP) based 5G technology.**

***Index Terms*—Real-time Analog Processing (RAP), Hilbert Transformer (HT), coupled loop resonator.**

I. Introduction

Real-time analog processing (RAP) has recently attracted significant interest, due to its inherent higher speed, lower cost and smaller power consumption features compared to purely conventional digital processing [1]–[4], and represents a most promising approach to 5G technology [5].

The deployment of RAP in radio technology naturally requires the availability of efficient analog-operator components, such as differentiators [6], integrators [7], expanders and compressors [8], time reversers [9], Fourier transformer [10] and Hilbert transformers [11].

The HT has been the less studied among these operators. Yet, it may find a diversity of applications, such as single side-band (SSB) modulation [7], bandpass and bandstop filtering [11] and edge detection [12]. Unfortunately, little details on their fundamental properties have been given in the HTs reported to date in the literature.

In this paper, we explain the subtleties of coupled loop resonator HTs. Specifically, we point out the relationships existing between the coupling coefficient, the phase response and the phase transition bandwidth, and indicate related design trade-offs.

II. Hilbert Transformer Principle

An HT is essentially a linear allpass device exhibiting a π phase rotation at its center frequency, as shown in Fig. 1. Figure 1(a) shows the response of an ideal HT, where the center frequency is zero and the phase slopes on either side of the frequency origin are zero. Such a response cannot be implemented practically, due to the time delays associated with propagation and to the progressiveness of the phase transition required by causality. However, the modified HT response shown in Fig. 1(b), with nonzero center frequency ω_0 and progressive phase transition, is perfectly realizable while still enabling typical HT applications.

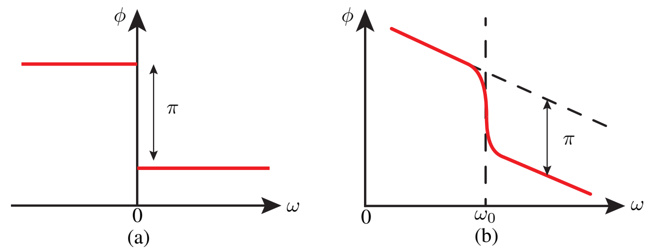

Fig. 1. Phase response of a Hilbert transformer (HT). (a) Ideal case. (b) Physical realization.

III. Coupled Loop Resonator Implementation

Figure 2 shows the schematic of a HT, that may be easily realized at both microwave and optical frequencies with a response of the type plotted in Fig. 1. This HT is composed of a 90° hybrid coupler with its isolated and coupled ports interconnected by a transmission line loop. Denoting the

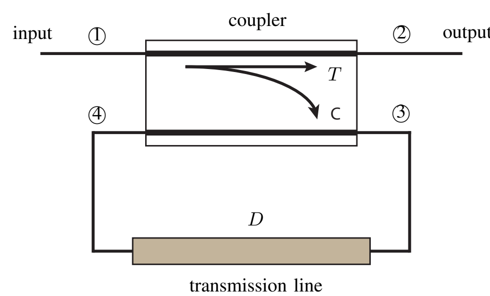

Fig. 2. Schematic of the proposed HT.

transmission and coupling coefficients of the coupler T and C, respectively, and the transmission response of the delay line D, the transfer function of the HT is expressed as

$$S_{21} = T + \frac{C^2 D}{1 - TD}, \tag{1}$$

where T, C and D are complex quantities.

Assuming, as a starting point a lossless coupler, power conservation demands

$$|T|^2 + |C|^2 = 1. \tag{2}$$

In the case of a quadrature coupler [$\angle(T, C) = \pi/2$], the phases of T and C are typically $-\pi$ and $-3/2\pi$, respectively, at the resonance frequency, ω_0. This, along with Eq. (2) and the assumption of linear phase response, leads to

$$T(\omega) = \sqrt{1 - |C|^2} e^{-j\frac{\omega}{\omega_0}\pi}, \tag{3}$$

$$C(\omega) = |C|e^{-j(\frac{\omega}{\omega_0}+\frac{1}{2})\pi}. \tag{4}$$

Substituting (3) and (4) into (1) yields an analytical expression for the transfer function of the system as a function of ω and $|C|$, $S_{21}(\omega; |C|)$, that we do not give here explicitly for the sake of conciseness. From this function, the phase and group delay responses are computed as

$$\phi(\omega; |C|) = \angle\{S_{21}(\omega; |C|)\} \tag{5a}$$

and

$$\tau(\omega; |C|) = -\frac{\partial\phi(\omega; |C|)}{\partial\omega}, \tag{5b}$$

respectively.

IV. Characterization and Explanation

Since the coupled part of the loop is $\lambda/2$-long, the shortest possible length of its uncoupled part is $3\lambda/2$-long, or $D = e^{-j\frac{3\omega}{\omega_0}\pi}$, corresponding to a 2λ resonant loop. Figures 3(a) and (b) plot the phase and group delay responses computed by (5a) and (5b) from (1), respectively. The responses in Fig. 3 may be

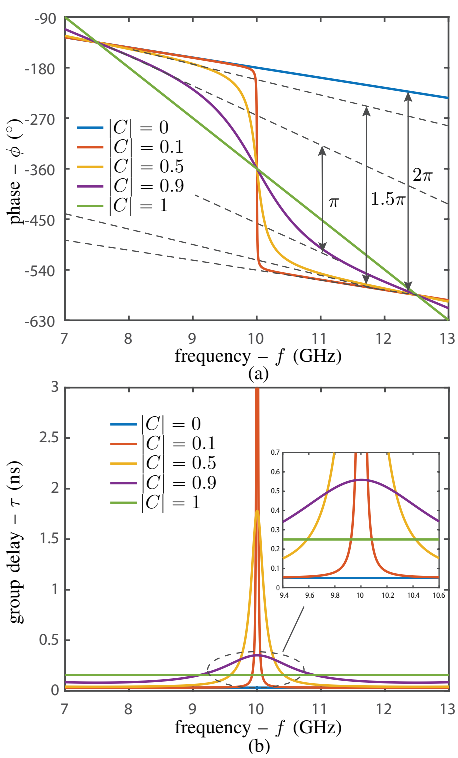

Fig. 3. Frequency response of the HT in Fig. 2 for different coupling coefficient magnitudes and $D = e^{-j\frac{3}{2}\pi}$. (a) Phase [Eq. (5a)]. (b) Group delay [Eq. (5b)].

understood as follows. In the limit case $C = 0$, the HT reduces to a simple transmission line, not seeing the loop, extending from port 1 to port 2, and therefore $S_{21}(\omega) = T(\omega) = e^{-j\frac{\omega}{\omega_0}\pi}$, with linear phase $\phi(\omega; 0) = -(\omega/\omega_0)\pi$ and therefore constant group delay $\tau(\omega; 0) = \pi/\omega_0$. In the other limit case $C = 1$, all the wave first experiences a $-\pi\omega/\omega_0$ phase shift, then gets entirely coupled into the loop where it experiences a phase shift of $\angle\{D\} = -3\pi\omega/\omega_0$, and finally acquires an additional phase shift of $-\pi\omega/\omega_0$, which leads to $\phi(\omega; 1) = -5\pi\omega/\omega_0$ and therefore constant group delay $\tau(\omega; 1) = 5\pi/\omega_0$. For $|C| \neq 1$, only a fraction of the energy is coupled into and out of the resonator, where the latter corresponds to energy loss from the viewpoint of the resonator; therefore, the higher $|C|$ is, the lower the Q of the resonator is, and hence the wider the resonance bandwidth is.

In fine, for $|C| \rightarrow 0$, we have a sharp phase transition, as required in an ideal HT, but the phase difference is 2π whereas the HT requires π. Reducing C allows one to reach the required phase difference of π, but this is at the cost of a larger phase transition bandwidth. Although the transition frequency band may not be a fundamental issues in some applications, we propose here the strategy of cascading two identical units of Fig. 2 for $|C| = 0.5$, which yields a $3\pi \equiv \pi$ phase difference with still a relatively sharp transition and also a relatively small coupling coefficient (3 dB), easily attainable in electromagnetic couplers.

V. Conclusion

We have characterized the coupled loop resonator Hilbert transformer and proposed an ideal implementation if in the form of a cascade of two units with $|C| = 0.5$. An experimental demonstration and applications of this HT will be presented at the conference.

References

[1] C. Caloz, S. Gupta, Q. Zhang, and B. Nikfal, "Analog signal processing: A possible alternative or complement to dominantly digital radio schemes," *IEEE Microw. Mag.*, vol. 14, no. 6, pp. 87 – 103, Sept. 2013.

[2] L. Zou, S. Gupta, and C. Caloz, "Real-time dispersion code multiple access (DCMA) for high-speed wireless communications," *arXiv*, vol. abs/1703.10516, 2017. [Online]. Available: http://arxiv.org/abs/1703.10516

[3] X. Wang, A. Akbarzadeh, L. Zou, and C. Caloz, "Flexible-resolution, arbitrary-input and tunable Rotman lens spectrum decomposer (RL-SD)," *arXiv preprint arXiv:1710.09226*, 2017.

[4] X. Wang, L. Zou, and C. Caloz, "Tunable C-section phaser for dynamic analog signal processing," in *Proc. XXXIInd URSI GASS*, Aug 2017, pp. 1–3.

[5] A. Osseiran, J. F. Monserrat, and P. Marsch, *5G mobile and wireless communications technology*. Cambridge University Press, 2016.

[6] H. V. Nguyen and C. Caloz, "First-and second-order differentiators based on coupled-line directional couplers," *IEEE Microw. Wirel. Compon. Lett.*, vol. 18, no. 12, pp. 791–793, 2008.

[7] W. Liu, M. Li, R. S. Guzzon, E. J. Norberg, E. J. Norberg, J. S. Parker, M. Lu, L. A. Coldren, and J. Yao, "A fully reconfigurable photonic integrated signal processor," *Nat. Photonics*, vol. 10, no. 2, pp. 190–195, Jan. 2016.

[8] S. Abielmona, S. Gupta, and C. Caloz, "Compressive receiver using a CRLH-based dispersive delay line for analog signal processing," *IEEE Trans. Microw. Theory Techn.*, vol. 57, no. 11, pp. 2617 – 2626, Nov. 2009.

[9] J. D. Schwartz, J. Azaña, and D. V. Plant, "An electronic temporal imaging system for compression and reversal of arbitrary UWB waveforms," in *Proc. IEEE Radio and Wireless Symp.*, Orlando, FL. U.S., Jan. 2008, pp. 487 – 490.

[10] M. A. G. Laso, T. Lopetegi, M. J. Erro, D. Benito, M. J. Garde, M. A. Muriel, M. Sorolla, and M. Guglielmi, "Real-time spectrum analysis in microstrip technology," *IEEE Trans. Microw. Theory Techn.*, vol. 51, no. 3, pp. 705–717, 2003.

[11] H. P. Bazargani, M. D. R. Fernández-Ruiz, and J. Azaña, "Tunable, nondispersive optical filter using photonic Hilbert transformation," *Opt. Lett.*, vol. 39, no. 17, pp. 5232–5235, Sept. 2014.

[12] J. A. Davis, D. E. McNamara, D. M. Cottrell, and J. Campos, "Image processing with the radial Hilbert transform: theory and experiments," *Opt. Lett.*, vol. 25, no. 2, pp. 99–101, Jan. 2000.