Experimental Validation of Super-Resolution Beamforming using Multiplicative Array Processing

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***Abstract*— Beamforming with conventional array processing utilizes linear, additive processing techniques to combine the signals from the different array elements. In previous work, a multiplicative processing technique was proposed for combining the signals from sensor arrays for super-resolution beamforming. The multiplicative processing technique is derived from standard linear array processing concepts and was shown to emulate the performance of larger array apertures. In this paper, experimental measurements from an acoustic sensor array are used to validate the proposed method for direction-of-arrival (DoA) estimation by comparing these results with those from linear processing of measurements from much larger aperture arrays. The processing results show that the multiplicative processing of experimental measurements from smaller apertures perform as well as linear processing of measurements from larger apertures.**

***Keywords*— multiplicative processing, array processing, beamforming, sensor arrays, super-resolution.**

I. INTRODUCTION

In array processing, when measurements from sensor arrays are combined to estimate the direction-of-arrival (DoA), the array measurements are generally processed using linear additive algorithms [1]. Multiplicative processing techniques, which combine the measurements from different array elements multiplicatively, were proposed several decades ago, but quickly discarded because of their perceived limitations [2-4]. In recent years, there has been renewed interest in multiplicative processing for beamforming in sensor arrays [5]. Multiplicative processing has the potential to produce enhanced angular resolution, which in turn improves the processor's capability to separate closely located sources.

In previous work from this project, a new multiplicative technique was shown to produce beamforming capability, which was commensurate with results from much larger arrays [6]. Unlike previous multiplicative processing techniques, the proposed method uses the product of multiple linearly combined signals from the same set of sensor array measurements. Multiple factors are generated from the same set of array measurements by combining them differently prior to multiplication. The proposed technique produces super-resolution capability, since the angular resolution is significantly finer than the angular resolution commensurate with the actual array size. In this follow-up paper, the multiplicative processing technique is applied to experimental measurements from an acoustic sensor array to verify and validate the technique.

II. MULTIPLICATIVE ARRAY PROCESSING

The proposed technique is derived from standard array processing analysis. The analysis assumes that the measurements are from a one-dimensional, N-element uniform linear array (ULA). Assuming that the incident wave is a plane wave in the far-field, the signal from a single source at the n^{th} element is described by $s_n = \rho_0 e^{j\phi_n}$, where the measurement phase is given by $\phi_n = k_0 x_n \sin\theta$. ρ_0 is the complex amplitude of the source, θ is the DoA angle and k_0 is the radar propagation constant. x_n is the position of the n^{th} sensor element. The different measurements from all N elements of the sensor array are represented in a single measurement vector as $\mathbf{s} = [s_1 \quad s_2 \quad \ldots \quad \ldots \quad s_N]^T$.

In standard processing, these measurements are combined linearly by projecting the measurement vector on to a steering vector, **a**, representing a particular direction of arrival (DoA) [1]. This projection is written as

$$r(\theta_i) = \mathbf{a}^T\mathbf{s} = \sum_{n=1}^{N} \rho_0 e^{jk_0 x_n[\sin\theta - \sin\theta_i]} , \qquad (1)$$

where θ_i is the DoA angle. By projecting the measurements over a set of steering vectors corresponding to a series of DoA angles, one generates an angular spectrum over these angles. Recognizing that (1) represents a polynomial, one may write (1) as

$$r(\theta_i) = \sum_{n=1}^{N} \rho_0 e^{jk_0 n\Delta(\sin\theta - \sin\theta_i)} = \rho_0 \sum_{n=1}^{N} A^n = \rho_0 \frac{1 - A^N}{1 - A} , \qquad (2)$$

where $A = e^{jk_0\Delta(\sin\theta - \sin\theta_i)}$ and Δ is the inter-element spacing. The polynomial may then be factorized into N-1 factors as

$$r(\theta_i) = \rho_0(1+\alpha_1 A)\,(1+\alpha_1 A)\,(1+\alpha_1 A)\times\ldots\times(1+\alpha_N A) \quad . \qquad (3)$$

Note each factor in (3) may be viewed as representing a linear combination of measurements from a sub-array of two elements. Equation (3) suggests that for a single source, the DoA spectrum from linear processing is equivalent to the DoA spectrum generated by multiplying *N*-1 factors, which represent *N*-1 paired sub-arrays. Note (3) represents the most primitive factors of the polynomial. Several of these factors may be re-combined to produce larger sub-arrays represented by higher-order polynomial factors. A similar polynomial factorization was used by Mittra et al [7], even though their processing was entirely linear and the implicit multiplication was implemented by the transmit/receive arrays.

III. Experimental Results and Discussion

Measurements from a 21-element acoustic array are used to validate the proposed multiplicative processing technique. The spacing between elements is designed as 0.076m (3 inches). A 2.3kHz source located 8.2° from broadside transmits a pure tone which is sampled at 44.1kHz at the receiver elements. Measurements from the 21-element array are used with standard linear processing and multiplicative processing with different sub-array sizes.

Fig. 1 shows the DoA spectra generated using linear processing with a 16-element array and multiplicative processing with a 2-element array. In the multiplicative processing approach, 15 factors are generated from a single measurement sample of a 2-element array and multiplied to produce the array pattern. This is equivalent to linear processing of measurements from a 16-element array and translates to a processing gain of 8. The results show a small error in the DoA estimate due to phase errors in the multiplicative processing approach.

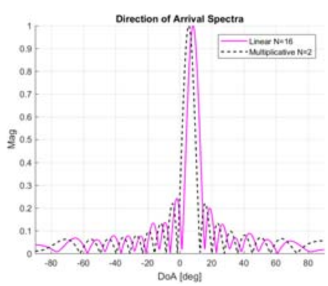

Fig 1 – DoA spectra from linear and multiplicative processing of measurements from a 16-element sub-array and a 2-element sub-array, respectively.

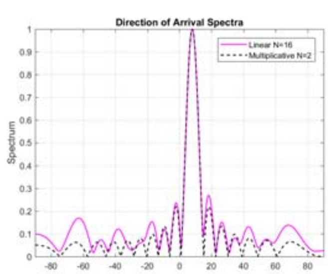

Fig. 2 – DoA spectra from linear and multiplicative processing of frequency domain measurements from a 16-element sub-array and a 2-element sub-array, respectively. Frequency domain processing is equivalent to averaging in the time domain and produces better DoA estimation compared to fig. 1.

In this study, averaging is achieved by using frequency domain processing. Since the transmitted signal is a 2.1kHz tone sampled at 44.1kHz, the time samples from each element are transformed into the frequency domain by using the FFT algorithm. The 2.1kHz frequency sample is selected from the output of the FFT and used in the linear and multiplicative processing methods. Fig. 2 shows the DoA spectra generated using linear processing with a 16-element array and multiplicative processing with a 2-element array. Since the frequency domain signal is the 2.1kHz tone averaged over several periods, the DoA estimates of the two approaches are almost identical with little or no error in the DoA estimation.

Fig. 3 shows frequency domain, linear and mulitplicative processing with a processing gain of 4. The multiplicative processing is performed using 5 factors of fourth order polynomials consisting of measurements from a single 4-element sub-array. The two spectra are very similar with nearly identical DoA estimates.

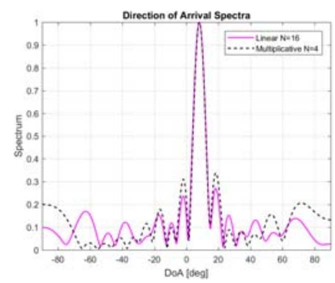

Fig 3 – DoA spectra from linear and multiplicative processing of measurements from a 16-element sub-array and a 4-element sub-array, respectively.

IV. Conclusions

In this follow up paper, the multiplicative processing developed for DoA estimation using multi-element sensor measurements is verified and validated using experimental measurements from an acoustic array. The results indicate that multiplicative processing of measurements from a small array produces angular resolution equivalent with linear processing of a much larger array. The results also indicate that frequency domain processing achieves better DoA estimation than processing single sample time-domain measurements.

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