Analytical Ray Tracing for Flat Lenses

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***Abstract*—An effective technique for the analysis of metasurfaces (MTSs) imposing boundary conditions that exhibit an axially symmetric variation is proposed in this work. It is shown that when the MTS exhibits a radial variability, the ray paths can be found in a very simple form and our technique provides a fast, yet accurate method to evaluate the trajectories of the rays propagating along both isotropic and anisotropic MTSs, either covered or uncovered by a metal plate. This allows for the efficient design of beam-forming devices.**

Keywords—metasurface, ray tracing, beam forming network

I. INTRODUCTION

Metasurfaces (MTSs) [1]-[2] are engineered surfaces constituted by a dense periodic texture of sub-wavelength elements printed on a dielectric slab, which can be macroscopically described through homogenized impedance boundary conditions (IBCs). Different sizes and shapes of the metallic elements implement different values of the surface impedance; therefore, media with variable propagation and dispersion properties can be engineered. In particular, MTSs typically support the propagation of surface waves (SWs), and the spatial variability of the IBC imposes a deformation of the SW wavefront, which addresses the local wavevector along non-rectilinear paths. This allows us to design planar devices with variable equivalent refractive index, such as lenses and beam forming networks to be used in antenna systems, with relatively low complexity [3]-[8]. The new possibilities offered by employing MTSs for such devices create the need for a tool that is able to characterize the propagation of SWs along these complex media.

II. FORMULATION FOR ROTATIONAL SYMMETRIES

Starting from the fundamentals of Transformation Optics [9]-[10], a series of studies were conducted towards a formulation for flat materials [11]-[14] which led to the general Flat Optics theory [15], where ray tracing, transport of energy, and ray velocity are rigorously derived for both cases of isotropic and anisotropic IBCs.

When the shape of the constituent elements of the MTS is regular enough, the impedance is a scalar and the effect of the IBC is isotropic with respect to the propagation direction of the SW. When the shape contains asymmetric features, like slots, grooves or cuts, the impedance is a tensor, thus introducing a characterization similar to anisotropy in volumetric media. This anisotropy can be exploited to extend the design capability in practical MTS-based devices [16]-[17].

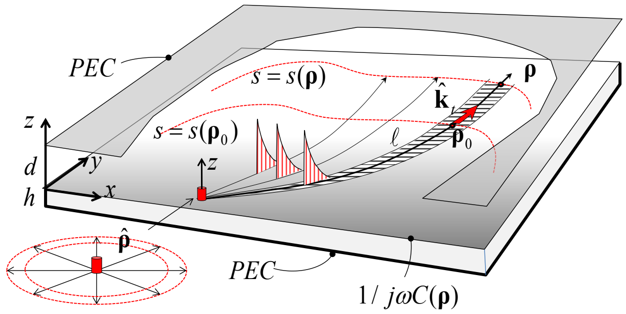

Fig. 1. Curved ray paths for a SW launched by a dipole source on an isotropic MTS with modulated reactance $X(\boldsymbol{\rho})$ inside a parallel plate waveguide

In this formulation, we consider a Cartesian reference system (x,y,z) with unit vectors $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$, and a parallel plate waveguide constituted by an impenetrable, scalar, continuous, lossless IBC imposed on a planar surface on the plane z=0, excited by a vertical dipole source, as depicted in Fig. 1.

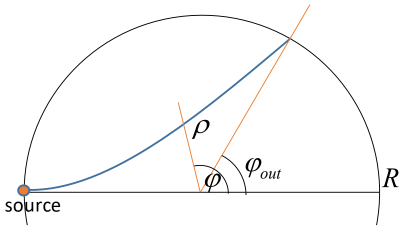

Fig. 2. Reference system for the ray paths.

The ray paths are subjected to an eikonal equation analogous to the one for Geometrical Optics rays in graded index materials

$$\nabla_t s \cdot \nabla_t s = n_{eq}^2\left(\boldsymbol{\rho}\right) \qquad (1)$$

where ∇_t denotes the transverse-to-z gradient. When the IBC distribution has a circular symmetry (i.e., the equivalent refractive index has a radial dependence only), the ray paths can be evaluated in closed form. If we consider the case $n_{eq}(\boldsymbol{\rho}) = n_{eq}(\rho)$, the eikonal equation, after some algebraic manipulations, leads to the expressions in eq. (2) for the ray trajectories, if using the reference system depicted in Fig. 2.

$$
\begin{aligned}
&\varphi(\rho, L) = \pi - \int_{\rho}^{R} \frac{L}{\rho' \sqrt{\left(\rho' n_{eq}(\rho')\right)^2 - L^2}} d\rho' \qquad \varphi \geq \varphi(\rho_{\min}, L) \\
&\varphi(\rho, L) = \varphi_{out} + \int_{\rho}^{R} \frac{L}{\rho' \sqrt{\left(\rho' n_{eq}(\rho')\right)^2 - L^2}} d\rho' \quad \varphi < \varphi(\rho_{\min}, L) \\
&\varphi_{out} = \pi - 2 \int_{\rho_{\min}}^{R} \frac{L}{\rho' \sqrt{\left(\rho' n_{eq}(\rho')\right)^2 - L^2}} d\rho'; \quad \varphi_{out} \in [-\pi, \pi]
\end{aligned}
\tag{2}
$$

Unlike standard numerical methods for evaluating the ray paths, where the trajectories need to be computed starting from the source and propagating according to the medium refractive index distribution, with the proposed technique the calculation of the entire trajectory may not be needed and its evaluation can be performed only in the points of interest (e.g. the boundary of the lens).

III. EXAMPLES

The proposed expressions for the ray paths have been verified numerically. Fig. 3 shows examples of ray trajectories calculated for different distributions of equivalent refractive index across the structure, namely, Generalized Maxwell's Fish-Eye (GMFE) lens [18] and a modified version of it, which produces a beam splitting effect.

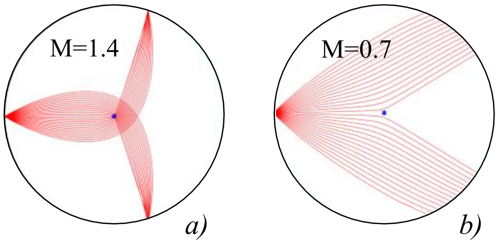

Fig. 3. Examples of Generalized Maxwell's fish-eye (GMFE) lense with M = 1.4 (left) and modified MFE (beam splitter, right) with M = 0.7.

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