A Periodic Array of Inhomogeneous Dielectric-Loaded Slots in an Infinite Thick Metallic Screen

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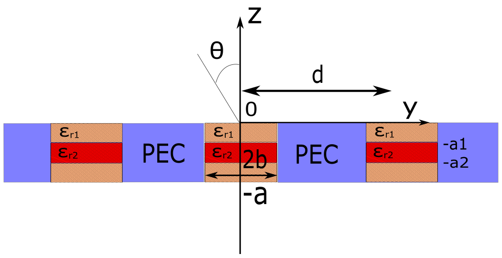

Fig. 1. A metallic shield with an inhomogeneously filled slot.

***Abstract*—Extraordinary transmission through a periodic array of inhomogeneous dielectric loaded slots in a an infinite metallic shield of finite thickness is demonstrated. We show that the intervals between those transmissions can be controlled by filled slots with inhomogeneous dielectric permittivity.**

I. Introduction

Metasurfaces [1] are well known for their promising applications. One kind of metasurface consists of a perforated metallic screen whose hole sizes are small compared to the free space wavelength. The studies of transmission through a single slot in an infinite thick metallic shield [2]-[8] reveal that extraordinary transmission is possible, and the spacing among those transmissions is controllable by introducing a region of different dielectric permittivity within a single slot [9]. As a natural extension of our previous work [10], an array of inhomogeneous dielectric loaded slots is investigated in present study, which confirms our expectation that what occurs for a single slot does so for the array structure as well: the intervals between extraordinary transmissions in a slot array can be controlled by introducing a region of different dielectric permittivity within the slots.

II. Derivation

Let an H-polarized electromagnetic wave be obliquely incident at an angle θ to a periodic array of inhomogeneous dielectric-loaded slots of width $2b$ in a perfectly conducting metallic shield of finite thickness a as shown in Figure 1. Each slot consists of three layers of different thicknesses, where the 1st and 3rd layers of thickness a_1 and $a_3 - a_2$ respectively are filled by a medium with relative permittivity ε_{r1}, while the layer in the middle of thickness $a_2 - a_1$ is filled by a medium with ε_{r2}. We assume for simplicity that the gap is symmetrically located ($a_3 - a_2 = a_1$) and that $\varepsilon_{r1} > \varepsilon_{r2}$. We start from an Floquet-mode formulation as used in [10] together with a mode-series expansion as used in [9]. The magnetic field is expressed as:

$$
\begin{aligned}
&H_x(y,z) = \\
&\quad a_i e^{-ik(\alpha y - \gamma z)} + \sum_{n=-\infty}^{\infty} a_n e^{-i(k_n y + \Lambda_n z)}, \\
&\quad (z > 0) \\
&\quad \sum_{m=0}^{\infty} [A_m e^{-ih_m(z+a_1)} + B_m e^{ih_m z}] \cos\frac{\pi m u}{2b}, \\
&\quad (-a_1 < z < 0) \\
&\quad \sum_{m=0}^{\infty} [C_m e^{-ig_m(z+a_2)} + D_m e^{ig_m(z+a_1)}] \cos\frac{\pi m u}{2b}, \\
&\quad (-a_2 < z < -a_1) \\
&\quad \sum_{m=0}^{\infty} [E_m e^{-ih_m(z+a_3)} + F_m e^{ih_m(z+a_2)}] \cos\frac{\pi m u}{2b}, \\
&\quad (-a_3 < z < -a_2) \\
&\quad \sum_{n=-\infty}^{\infty} d_n e^{-i(k_n y - \Lambda_n (z+a))}, \ (z < -a_3)
\end{aligned}
\tag{1}
$$

where a_i is amplitude of incident field, a_n and d_n are Floquet-Bloch mode amplitudes, $\alpha = \sin\theta$, $\gamma = \cos\theta$ (θ being the angle of incidence), $k_n = k\alpha + \frac{2\pi n}{d}$, $\Lambda_n = \sqrt{k^2 - k_n^2}$, $u = y + b$, $h_m = \sqrt{k^2\varepsilon_{r1} - (\frac{\pi m}{2b})^2}$, $g_m = \sqrt{k^2\varepsilon_{r2} - (\frac{\pi m}{2b})^2}$ and A_m, B_m, C_m, D_m , E_m and F_m are amplitudes of parallel-plate waveguide mode m in the various regions of the slot, related to reflection and transmission coefficients at the interfaces. As in [9], we can obtain A_m, C_m, D_m and F_m in terms of B_m and E_m by applying the boundary conditions of continuity of H_x and $\frac{1}{\tilde{k}^2}\frac{\partial H_x}{\partial z}$ (where $\tilde{k} = k\sqrt{\varepsilon_r}$) at the interfaces between the dielectrics within the slot $|y| < b$ at $z = -a_1$ and $z = -a_2$. Next, as in [7]-[8], we apply the boundary condition $\frac{\partial H_x}{\partial z} = 0$ at the surface of the metallic shield and continuity of H_x and $\frac{1}{\tilde{k}^2}\frac{\partial H_x}{\partial z}$ at the slot surfaces ($z = 0$ and $z = -a_1$). Defining $\tilde{x}_q^{\pm} = x_q^{\pm}\Lambda_q' e^{-ik\alpha y}$, $\Lambda_n = k\gamma\Lambda_n'$, $x_n^{\pm} = \tilde{a_n} \pm d_n$ and $\tilde{a_n} = a_n$ if $n \neq 0$ and $\tilde{a_0} = a_0 - 2a_i$ if $n = 0$, we arrive at the integral

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equations:

$$\tilde{x}_n^{\pm} = \frac{16\pi k\alpha b^3}{k\gamma\varepsilon_r d^2} n \sum_{m=0}^{\infty} \frac{h_m \Gamma_m^{\mp}}{\Gamma_m^{\pm}(1+\delta_{0m})} G_m\left(\frac{2\pi bn}{d}\right) G_m(kb\alpha)$$
$$+ \frac{8\pi b^3 n e^{ik\alpha y}}{k\gamma\varepsilon_r d^2} \sum_{m=0}^{\infty} \frac{h_m \Gamma_m^{\mp} G_m\left(\frac{2\pi bn}{d}\right)}{\Gamma_m^{\pm}(1+\delta_{0m})} \sum_{q=-\infty}^{\infty} \frac{\tilde{x}_q^{\pm}}{\Lambda_q'} k_q G_m(k_q b) \quad (2)$$

where

$$G_m(x) = \frac{\sin(x - \frac{\pi m}{2})}{(x)^2 - (\frac{\pi m}{2})^2}, \quad \Gamma_m^{\pm} = \frac{\Gamma_{2m}^{\pm}}{\Gamma_{1m}^{\pm}}$$

$\Gamma_{1m}^{\pm} = \frac{e^{-i2h_m a_1}}{T_1+T_2} \pm 1 \mp \frac{T_3 e^{-i2h_m a_1}}{T_1+T_2}, \quad \Gamma_{2m}^{\pm} = \frac{e^{-i2h_m a_1}}{T_1+T_2} \mp 1 \mp \frac{T_3 e^{-i2h_m a_1}}{T_1+T_2}$

$T_{1m} = \cos g_m(a_2 - a_1)$, $T_{2m} = \frac{i}{2}\left[t_m + \frac{1}{t_m}\right]\sin g_m(a_2 - a_1)$

$T_{3m} = \frac{i}{2}\left[t_m - \frac{1}{t_m}\right]\sin g_m(a_2 - a_1)$, $t = \frac{g_m \varepsilon_{r1}}{h_m \varepsilon_{r2}}$.

We assume normal incidence ($\theta = 0$), and further that $kb\sqrt{\varepsilon_r} < \frac{\pi}{2}$; the latter condition ensures that all other modes will be evanescent except m =0 (TEM mode). We extract the TEM mode ($m = 0$) terms in (2) and get a degenerate-kernel integral equation whose solution is:

$$\tilde{x}_n^{\pm} = \frac{4b}{d\sqrt{\varepsilon_{r1}}} \operatorname{sinc}\left(\frac{2\pi bn}{d}\right) N^{\pm} \quad (3)$$

where

$$N^{\pm} = \frac{\frac{\Gamma_0^{\mp}}{\Gamma_0^{\pm}}}{1 - \frac{d}{2\pi^2 b\sqrt{\varepsilon_{r1}}} \frac{\Gamma_0^{\mp}}{\Gamma_0^{\pm}} I_{00}}$$

For $kd \ll 2\pi$, $I_{00} = \left(\frac{2b\pi}{d}\right)^2 + \frac{jkd}{2\pi}\left[\mathrm{Cl}_3(0) - \mathrm{Cl}_3\left(\frac{2b\pi}{d}\right)\right]$ where Cl_3 is third-order Clausen function [11]. From (3), we can determine the magnitude of the plane-wave transmission coefficient:

$$|S_{12}| = \left|\frac{H_{xt}(y,z)}{H_{xi}(y,z)}\right| = \left|\frac{2b}{d\sqrt{\varepsilon_r}}[N^+ - N^-]\right| \quad (4)$$

III. Results

Extraordinary transmisssion through both a single and an array of dielectric-loaded slots in thick metallic shield has been studied previously [7]-[10]. It has been shown [9] that a region of different dielectric permittivity introduced within a single slot will control the intervals between resonances, a result we now extend to the slot array. A comparison among results of the inhomogeneous (gap) formula (4), the homogeneous slot (no gap) formula from [10] and a full wave HFSS simulation is shown in Figure 2. The effect of slot inhomogeneity for an array is similar to that of a single slot [9],—introducing a gap within the slots shifts the even-order resonances to higher frequencies while keeping odd-order resonances almost unchanged. Increasing the size of the gap will shift the even order resonances to progressively higher frequencies until they meet the next odd-order resonances, which will be shifted to higher frequencies but at a rate much slower than for the even order resonances, eventually forming new resonances which will keep moving to higher frequencies as the size of the gap is increased.

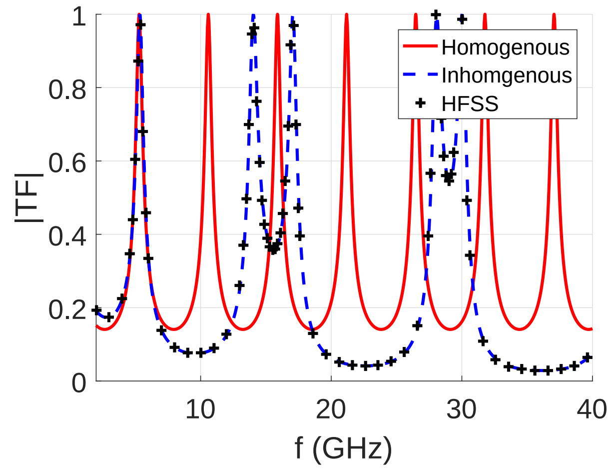

Fig. 2. Transmission Factor TF of homogeneous and inhomogeneous symmetrically filled ($a_3 - a_2 = a_1$) slots: $a_3 = 4$ mm, $b = 1$ mm, $\varepsilon_{r1} = 50$, $\varepsilon_{r2} = 1$ (air) and gap= 1 mm.

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