Target Detection in Phased-MIMO Radars with Two Detectors

Safieh Jebali, Hengameh Keshavarz
Department of Communication Engineering
University of Sistan and Baluchestan
Zahedan, Iran
Email: Safieh.Jebali@yahoo.com, hkeshavarz@ieee.org

***Abstract*—In this paper, the problem of target detection in phased-MIMO radars is considered and target detection performance of phased-MIMO radars is compared with MIMO and phased-array radars. Phased-MIMO radars combine advantages of the MIMO and phased-array radars. In these radars, the transmit array will be partitioned into a number of subarrays that are allowed to overlap and each subarray transmits a waveform which is orthogonal to the waveform transmitted by other subarrays. In this paper, target detection performance of phased-MIMO radars is analyzed with two detectors by both analytical and simulation results. It is assumed that the transmitted waveforms are ideally orthogonal. The Generalized likelihood ratio test (GLRT) and the likelihood ratio test (LRT) are used for target detection. The closed-form expressions of the false alarm and detection probability in presence of Gaussian noise are obtained. Simulation results validate the theoretical analysis.**

I. Introduction

In the last decade, multiple-input multiple-output radars have become the focus of attention of researchers [1]-[2]. Based on antenna configurations, MIMO radars can be classified into two types. The first type is known the statistical/distributed MIMO radars, where its antennas are separated far from each other such that a target can be viewed from different spatial directions to achieve spatial diversity gain [1], [2]. The second type is known the co-located MIMO radars, where the transmitter and receiver antennas are closely spaced to transmit a beam towards a certain direction in the space [3], [4]. Recently, adding the Phased-Array radars to the MIMO radars with co-located antennas has been called phased-MIMO radars. The essence of this technique is on partitioning the transmitting array to a number of overlapped subarrays with smaller sizes such that each subarray operates in the phased-array mode. Hence, phased-MIMO radars exploit jointly the benefits of the phased-array and MIMO radars [5]-[8]. In this paper, target detection is analyzed with GLRT and LRT detectors in phased-MIMO radars, when all subarrays transmit orthogonal waveforms. Furthermore, a comparison is made among target detection in phased-array, MIMO and phased-MIMO radars. This paper is organized as following: firstly, the signal model phased-MIMO radar is presented in Section II. Section III presents the problem of target detection in phased-MIMO radars with orthogonal transmitted waveforms. In Section IV, simulation results are shown. Finally, Section V concludes the paper.

II. Signal model for phased-MIMO radar

In the phased-MIMO radars, the transmit array is partitioned into K subarrays $(1 \leq K \leq M_t)$ overlapping such that no subarray is exactly the same as another subarray. The signal reflected by the target is given by:

$$\mathbf{r}(t,\theta) = \sqrt{\frac{M_t}{K}}\beta(\theta)(\mathbf{c}(\theta) \odot \mathbf{d}(\theta))\Phi_k(t) \tag{1}$$

where $\mathbf{c}(\theta)$, $\mathbf{d}(\theta)$ and $\mathbf{a}(\theta)$ are the transmit coherent processing vector, waveform diversity vector, and actual transmit steering vector [5]-[8], respectively; $\beta(\theta)$ is the target reflection coefficient, $\tau_K(\theta)$ is the time delay for the signal to travel between the first antenna of the kth subarray and the first antenna of the transmit array, and $\odot$ denotes the Hadamard product operator. Hence, the received vector of array observations is written as:

$$\mathbf{X}(t) = \mathbf{r}(t,\theta_s)\mathbf{b}(\theta_s) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{n}(t)$ and $\mathbf{b}(\theta_s)$ are the noise term and the actual receive steering vector associated with direction θ_s [5]-[8], respectively. The virtual data vector after matched-filtering is converted into:

$$\mathbf{y} = \sqrt{\frac{M_t}{K}}\beta_s\mathbf{u}(\theta_s) + \mathbf{n} \tag{3}$$

where n is the noise term with covariance matrix $R_n = \sigma_n^2\mathbf{I}_{KM_r}$ and σ_n^2 is noise power, and $\mathbf{u}(\theta) \triangleq (\mathbf{c}(\theta) \odot \mathbf{d}(\theta)) \otimes \mathbf{b}(\theta)$ is the virtual steering vector associated with direction θ [5]-[8].

III. Target Detection

In this section, two detectors, namely the LRT and GLRT detectors, are presented. The optimal detector in the Neyman-Pearson criterion is the likelihood ratio test [4].

A. The GLRT detector

The radar detection problem is formulated as: $\begin{cases} H_0 : \mathbf{y} = \mathbf{n} \\ H_1 : \mathbf{y} = \sqrt{\frac{M_t}{K}}\alpha + \mathbf{n} \end{cases}$, where $\mathbf{y}$ is a output vector of the each matched filter at every receiver, $\mathbf{n}$ is a white Gaussian noise vector and α is defined as $\alpha = \beta_s\mathbf{a}(\theta_s)\otimes\mathbf{b}(\theta_s)$ for MIMO radar and $\alpha = \beta_s(\mathbf{c}(\theta) \odot \mathbf{d}(\theta)) \otimes \mathbf{b}(\theta)$ for Phased-MIMO radar. so in this detection problem, generalized likelihood ratio test (GLRT) can be employed by replacing the unknown coefficient vector α by its ML estimate then the likelihood ratio test can be written as $\frac{max_\alpha p(\mathbf{y}|H_1,\sigma_n^2,\alpha)}{p(\mathbf{y}|H_0,\sigma_n^2)} \gtrless_{H_0}^{H_1} T$ Hence, the log likelihood ratio can be written as:

$$\ln(\frac{p(\mathbf{y}|H_1,\sigma_n^2,\widehat{\alpha_{ML}})}{p(\mathbf{y}|H_0,\sigma_n^2)}) = \frac{\mathbf{y}^H\mathbf{y}}{\sigma_n^2} \tag{4}$$

and the likelihood ratio test becomes $\|\mathbf{y}\|^2 \gtrless_{H_0}^{H_1} T^{'}$, where $T^{'}$ is the accordingly modified version of T and $\|.\|$ represents the Fobenious norm.

B. The LRT detector

Based on [4], the radar detection problem is formulated as $\begin{cases} H_0 \text{ :target does not exist} \\ H_1 \text{ :target exists} \end{cases}$. The optimal detector in the Neyman-Pearson criterion is the likelihood ratio test [4] and its log-form is defined as $T = \log \frac{p_y(\mathbf{y}|H_1)}{p_y(\mathbf{y}|H_0)}) \gtrless_{H_0}^{H_1} \xi$. where ξ refers to the threshold and is equivalent to the desired probability of the false alarm. The threshold is calculated by:

$$T = -\frac{1}{2\sigma_n^2}y_1^H y_1 + \frac{1}{2\sigma_n^2}y_0^H y_0 + \frac{(\mathbf{U}^H(\theta)y_1)}{\sigma_n^2} - \frac{(\mathbf{U}^H(\theta)y_0)}{\sigma_n^2} \tag{5}$$

Hence, the new detector is defined as $\eta = \mathbf{U}^H(\theta)\mathbf{y}$, and the optimal detector is given by $\eta \gtrless \xi^{'}$, where $\xi^{'}$ is the new threshold. Since, the equation of radar system Gaussian noise was modeled by $\mathbf{n} \sim N(0,\sigma_n^2)$. According to the distribution of the test statistic, $\frac{\eta}{\sqrt{\sigma_n^2\mathbf{U}^H(\theta)\mathbf{U}(\theta)}} = \eta^{'}$, and the threshold is replaced by $\xi^{''} = \xi^{'}/\sqrt{\sigma_n^2\mathbf{U}^H(\theta)\mathbf{U}(\theta)}$. Finally, the probability of the false alarm is given by:

$$P_{FA} = P(H_1;H_0) = P_r\{\eta^{'} > \xi^{''};H_0\} = Q(\xi^{''}) \tag{6}$$

and $\xi^{''} = Q^{-1}(P_{FA})$, where $Q(.)$ and $Q^{-1}(.)$, denote the cumulative distribution function of the normal distribution and the inverse cumulative distribution function of the standard normal distribution, respectively. The detection probability is defined as:

$$\begin{aligned} P_D &= P(H_1;H_1) = P_r\{\eta^{'} > \xi^{''};H_1\} \\ &= Q(\xi^{''} - \sqrt{\mathbf{U}^H(\theta)\mathbf{U}(\theta)/\sigma_n^2}) \end{aligned} \tag{7}$$

IV. SIMULATION RESULTS

In this section, simulations results are represented to compare the performances of target detection in phased-MIMO radars, phased-array radars and co-located MIMO radars. It is assumed that transmitted waveforms are ideally orthogonal. Numerical results shown in this section are obtained by $10,000$ Monte Carlo simulation runs.

V. CONCLUSION

Phased-MIMO radars are recently introduced in the literature in order to improve parameter estimation capability of co-located MIMO radars. However, target detection performance of these radars has not been investigated yet. In this paper, the GLRT and LRT target detector are considered and target detection performance is analysed in phased-MIMO radars transmitting fully orthogonal waveforms. Simulation results show that detection performance in the phased-MIMO radars is higher than the phased-array radars and lower than the MIMO radars with co-located antennas. Moreover, increasing the false alarm probability, correlation coefficients, and number of subarrays leads to growing target detection performance in the phased-MIMO radars. It can be seen that with the same probability of the false alarm, the probability of detection in LRT detectors is higher than GLRT detectors.

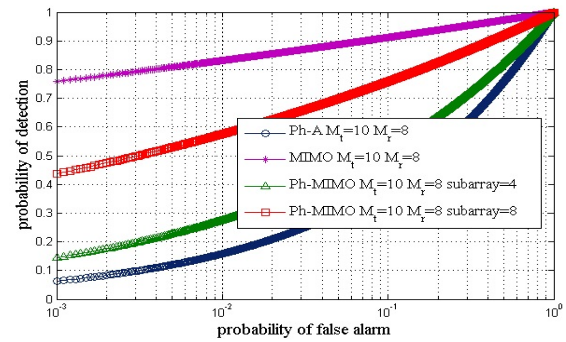

Fig. 1. Comparison of receiver-operating-characteristic (ROC) curves in GLRT detectors with $M_t = 10$, $M_r = 8$.

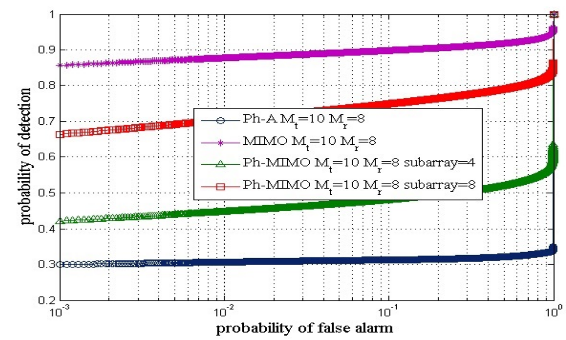

Fig. 2. Comparison of receiver-operating-characteristic (ROC) curves in LRT detectors with $M_t = 10$, $M_r = 8$.

REFERENCES

[1] A. Haimovich, R. Blum, and L. Cimini, "MIMO radar with widely separated antennas", *IEEE Signal Process*. Mag., vol. 25, pp. 116–1 29, Jan. 2008.

[2] V. Chernyak, "On the Concept of MIMO Radar", *IEEE Radar Conference 2010.*

[3] J. Li and P. Stoica, "MIMO radar with colocated antennas",*IEEE Signal Process. Mag.*, vol. 24, pp. 106–114, Sep. 2007.

[4] Z. Qiu, J. Yang, H. chen, X. Li and Z. Zhuang, "Effects of transmitting correlated waveforms for co-located MIMO radar with target detection and localisation", *IET Signal Process*, vol. 7, Iss. 9, pp. 897–910, 2013.

[5] A. Hassanien and S. A. Vorobyov, "Phased-MIMO radar: A tradeoff between phased-array and MIMO radars", *IEEE Trans. Signal Processing*, vol. 58, pp. 3137–3151, Jun. 2010.

[6] A. Hassanien and S. A. Vorobyov, "Why the phased- MIMO radar outperforms the phased-array and MIMO radars", *Proc. 18th European Signal Processing Conf.*, Aalborg, Denmark, pp. 1234–1238, Aug. 2010.

[7] T. D. Backes, "Parameter Identifiability in a Phased-subarray MIMO" ,*IEEE Aerospace Conference*, Big Sky, MT, March 2014.

[8] W. Q. Wang, "Phased-MIMO Radar with Frequency Diversity for Range-Dependent Beamforming" ,*IEEE Sensors Journal*, vol. 13, no. 4, pp. 1320–1328, April 2013.