Waveform Design for Improved Doppler Accuracy

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***Abstract*—A waveform design procedure for improving the estimation of Doppler frequencies in active remote sensing applications is presented. The bound on frequency estimation is analyzed in terms of a continuous waveform, and the optimal waveform is inferred. Several waveform designs are analyzed, showing that a *near-optimal* dual-pulse waveform can achieve greater estimation accuracy than a single-pulse waveform using the same signal energy.**

I. Introduction

Accurate estimation of the Doppler frequency shift of a received signal is critically important in many remote sensing applications, including radar target tracking [1], synthetic aperture radar [2], underwater sonar [3], and medical ultrasound [4], among others. While the bounds on the ability to estimate the frequency of a signal are well-known, the efforts to improve the Doppler estimation accuracy of a measurement almost universally focus on the use of uniform-amplitude signals and analysis of the received signal properties [5]–[8]. Significant accuracy improvements can be achieved with such post-processing of the received signal, however this approach does not take into account the ability to design the transmitted waveform to achieve improved estimation accuracy.

In this work, we demonstrate a method of achieving improved Doppler frequency estimation by designing the transmitted signal waveform. By analyzing the properties of the continuous form of the lower bound on estimation accuracy, important insights into the design of the waveform can be gained; in particular, a dual-pulse waveform achieves greater frequency estimation performance relative to a uniform pulse of the same duration and energy. We further note that this approach is generally independent of other estimation improvement techniques such as post processing; thus, most techniques for accuracy improvements (e.g. [9], [10]) may still be applied for further improve the accuracy.

II. Accuracy of Doppler Frequency Estimation

A narrowband radar signal scattered off a point target that is moving radially relative to the radar can be modeled with as a signal with frequency shift proportional to the relative velocity. For a radially inbound object with velocity $v \ll c$, the received signal can be modeled by

$$s_r(t) = \alpha g(t - t_0) e^{j2\pi f_d t} + w(t) \quad (1)$$

where $g(t)$ is the transmitted signal, t_0 is the delay due to propagation, $f_d = 2\frac{f_c v}{c}$ is the Doppler frequency shift, f_c is the RF center frequency of the signal, the term α includes all the constant terms, and $w(t)$ is a white Gaussian noise term. The ability to measure the radial velocity is limited by the variance on the estimate of the frequency of (1). This variance is given for most narrowband signals by [11]

$$\text{var}(\hat{f}_d - f_d) \geq \frac{N_o}{2|\alpha|^2 \zeta_t^2} \quad (2)$$

where

$$\zeta_t^2 \triangleq \int (2\pi t)^2 |g(t)|^2 dt \quad (3)$$

is the mean-square time duration of the signal.

As indicated by (2) and (3), the accuracy of the Doppler frequency estimate is inversely proportional to the mean-square time duration, which is the second moment of the temporal signal. In comparison, Doppler resolution is inversely proportional to the radar signal temporal duration (e.g. [12]). Thus, to achieve the greatest Doppler accuracy, it is desirable to maximize the mean-square time duration of the signal, which is not equivalent to simply maximizing the temporal duration of the signal.

III. Optimal Waveform Design

Maximizing the the mean-square duration of the signal is equivalent to maximizing the temporal second moment as indicated by (3). For a given time duration T, the signal is maximized when all the energy is concentrated into two separate pulses of infinitesimal width. Mathematically, this can be given by

$$g_o(t) = \lim_{\tau \to 0} g(f_c, t) H(t, \mathrm{T}, \tau) \quad (4)$$

where

$$g(f_c, t) = A e^{j2\pi f_c t + \phi(t)} \quad (5)$$

is the base waveform with amplitude A, carrier frequency f_c, $\phi(t)$ is any phase modulation, and

$$H(t, \mathrm{T}, \tau) = \frac{1}{\tau} \left[h(t) - h(t - \tau) + h(t - \mathrm{T} + \tau) - h(t - \mathrm{T}) \right]. \quad (6)$$

In the limit as $\tau \to 0$, the two pulses become narrower, with the amplitude increasing (A/τ). Thus, the overall signal energy is preserved.

The optimal waveform consists of two infinitesimally short pulses of infinite amplitude however in practice the two pulses will have finite width τ and amplitude A/τ. In this section we analyze the relative performance of the waveform as a function of τ to determine the limits of a *near-optimal* waveform with finite parameters. The waveform was simulated with f_c = 1 GHz, T = 10 μs, and signal energy E_s = 1, which was consistent for all variations of τ. The waveform was analyzed for values of τ/T = 1, 0.5, 0.1, and 0.02. To include some

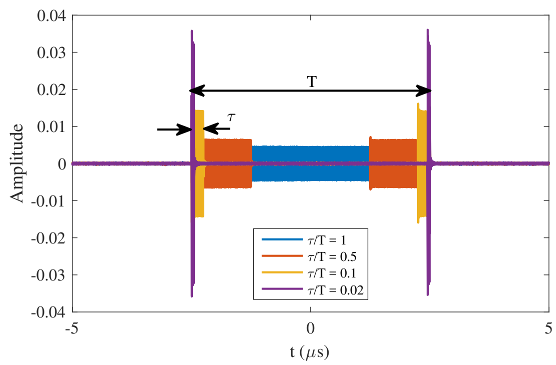

Fig. 1. Four different waveforms of progressively increasing second moment.

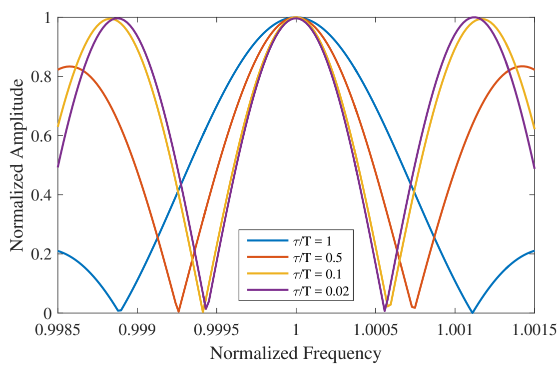

Fig. 2. Frequency domain of the four signals.

effects of the received system responses, the signals were filtered around f_c using a bandpass 3-pole Chebychev filter with a 200 MHz bandwidth. Fig. 1 shows the time-domain responses of the four waveforms with a 30 dB signal-to-noise ratio, showing clearly that as τ is decreased, the signal energy becomes compressed into the ends of the waveform, thereby increasing the second moment of the signal. The Fourier transform of the four 30 dB SNR signals is shown in Fig. 2. The frequency-domain response of the waveform becomes progressively narrower as τ decreases, thereby improving the frequency estimation. It can also be seen, however, that frequency ambiguities become more prominent as τ decreases; this represents an area where frequency disambiguation must be addressed. The estimation bounds for the waveforms are plotted in Fig. 3, showing the clear improvement of the near-optimal waveforms. Also shown for reference is the 3 dB resolution of the waveforms, which is only determined by the signal duration T and is consistent for all waveforms. The Doppler estimation accuracy improves as τ decreases, however the figures above show that up to a certain point the relative gains in estimation accuracy become limited. In particular, the differences in accuracy between $\tau/\mathrm{T} = 1$ and $\tau/\mathrm{T} = 0.5$, and that between $\tau/\mathrm{T} = 0.5$ and $\tau/\mathrm{T} = 0.1$ are significantly greater than that between $\tau/\mathrm{T} = 0.1$ and $\tau/\mathrm{T} = 0.02$. After a minimum $\tau/\mathrm{T} \approx 0.1$, the improvements minimally increase.

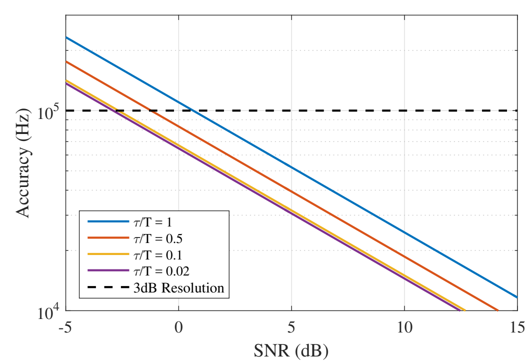

Fig. 3. Lower bounds on accuracy as a function of signal-to-noise ratio (SNR).

IV. Conclusion

By designing the transmit waveform, the Doppler frequency estimation accuracy can be improved over a simple uniform-amplitude waveform. In the limit, the optimal waveform consists of two temporal impulse signals; however in practice the best waveform may be a near-optimal design using $\tau/\mathrm{T} \approx 0.1$. In particular, reducing τ and increasing the pulse amplitudes will have implications on the transmitter and receiver hardware (e.g. for bandwidth and peak power handling) that must also be considered. Frequency ambiguities in near-optimal waveforms must also be addressed to ensure precise and accurate Doppler frequency measurement.

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