Influence of Inhomogeneity in a Dielectric-Loaded Slot in an Infinite Thick Metallic Shield on Higher-Order Mode Resonances.

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***Abstract*—Extraordinary transmission through an inhomogeneous dielectric loaded slot in a an infinite metallic shield of finite thickness is demonstrated. We show that the higher order (Fano) transmission resonances' locations and magnitudes can be controlled by introducing a region of different dielectric constant within the slot.**

I. Introduction

Many authors have studied transmission through an infinite dielectric-loaded slot in an infinite metallic shield of finite thickness, but none of them has included the higher order parallel-plate waveguide modes in their calculation, or considered inhomogeneous loading of the slot [1]-[7]. In our recent work [8]-[9], we have included these effects in our calculation, which revealed new resonances (Fano resonances) that result from including higher order modes, and the ability to control the intervals between resonances of the fundamental mode ofthe parallel-plate waveguide by introducing a region of different dielectric permittivity within the slot (inhomogenous case). In this paper, we extend our analysis to study the effect of introducing a region of different dielectric permittivity within the slot on the higher-order Fano resonances. Our study reveals that the thickness of the gap within the slot, as well as the dielectric constant of the substance that fills the gap, can control the location and magnitude of the higher-order resonances.

II. Derivation

Let an H-polarized electromagnetic wave be obliquely incident at an angle θ to a single inhomogeneous dielectric-loaded slot of width $2b$ in a perfectly conducting metallic shield of finite thickness a as shown in Figure 1. The slot consists of three layers of different thicknesses where the 1st and 3rd layers of thickness a_1 and $a_3 - a_2$ respectively are filled by a medium with relative permittivity ε_{r1}, while the layer in the middle of thickness $a_2 - a_1$ is filled by a medium with ε_{r2}. We assume for simplicity that the gap is symmetrically located ($a_3 - a_2 = a_1$) and that $\varepsilon_{r1} > \varepsilon_{r2}$. We start from an integral

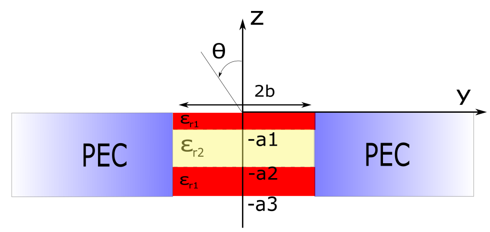

Fig. 1. A metallic shield with an inhomogeneously filled slot.

equation formulation with a mode-series expansion as used in [9]. The magnetic field is expressed as:

$$H_x(y,z) = \begin{cases} e^{-ik(\alpha y - \gamma z)} + e^{-ik(\alpha y + \gamma z)} + \int_{-\infty}^{\infty} a(\xi) e^{-ik(\sqrt{1-\xi^2} z + \xi y)}\, d\xi, & (z > 0) \\ \sum_{m=0}^{\infty} [A_m e^{-ih_m(z+a_1)} + B_m e^{ih_m z}] \cos \frac{\pi m u}{2b}, & (-a_1 < z < 0) \\ \sum_{m=0}^{\infty} [C_m e^{-ig_m(z+a_2)} + D_m e^{ig_m(z+a_1)}] \cos \frac{\pi m u}{2b}, & (-a_2 < z < -a_1) \\ \sum_{m=0}^{\infty} [E_m e^{-ih_m(z+a_3)} + F_m e^{ih_m(z+a_2)}] \cos \frac{\pi m u}{2b}, & (-a_3 < z < -a_2) \\ \int_{-\infty}^{\infty} d(\xi) e^{ik(\sqrt{1-\xi^2}(z+a_3) - \xi y)}\, d\xi, & (z < -a_3) \end{cases} \tag{1}$$

where $\alpha = \sin\theta$, $\gamma = \cos\theta$ (θ being the angle of incidence), $u = y + b$, $h_m = \sqrt{k^2\varepsilon_{r1} - (\frac{\pi m}{2b})^2}$, $g_m = \sqrt{k^2\varepsilon_{r2} - (\frac{\pi m}{2b})^2}$ and A_m, B_m, C_m, D_m , E_m and F_m are amplitudes of parallel-plate waveguide mode m in the various regions of the slot, related to reflection and transmission coefficients at the interfaces. As in [9], we can obtain A_m, C_m, D_m and F_m in terms of B_m and E_m by applying the boundary conditions of continuity of H_x and $\frac{1}{\tilde{k}^2}\frac{\partial H_x}{\partial z}$ (where $\tilde{k} = k\sqrt{\varepsilon_r}$) at the interfaces between the dielectrics within the slot $|y| < b$ at $z = -a_1$ and $z = -a_2$. Next, as in [8], we apply the boundary condition $\frac{\partial H_x}{\partial z} = 0$ at the surface of the metallic shield and continuity of H_x and $\frac{1}{k^2}\frac{\partial H_x}{\partial z}$ at the slot surfaces ($z = 0$ and $z = -a_1$). Defining $\tilde{x}^{\pm}(\zeta) = [a(\zeta) \pm d(\zeta)]\sqrt{1-\zeta^2}$, we arrive at the integral equations:

$$\tilde{x}^{\pm}(\zeta) = \frac{4\alpha(kb)^2\zeta}{\pi\varepsilon}\sum_{m=0}^{\infty}\frac{h_m b\Gamma_m^{\pm}}{(1+\delta_{0m})}G_m(\zeta)G_m(\alpha)$$
$$+\frac{2(kb)^2\zeta}{\pi\varepsilon}\sum_{m=0}^{\infty}\frac{h_m b\Gamma_m^{\pm}}{(1+\delta_{0m})}\int_{-\infty}^{\infty}\tilde{x}^{\pm}(\xi)\frac{\xi}{\sqrt{1-\xi^2}}G_m(\zeta)G_m(\xi)\,d\xi \quad (2)$$

where

$$G_m(\zeta) = \frac{\sin(kb\zeta - \frac{\pi m}{2})}{(kb\zeta)^2 - (\frac{\pi m}{2})^2}, \quad \Gamma_m^{\pm} = \frac{\Gamma_{2m}^{\pm}}{\Gamma_{1m}^{\pm}}$$

$$\Gamma_{1m}^{\pm} = \frac{e^{-i2h_m a_1}}{T_1+T_2} \pm 1 \mp \frac{T_3 e^{-i2h_m a_1}}{T_1+T_2}, \quad \Gamma_{2m}^{\pm} = \frac{e^{-i2h_m a_1}}{T_1+T_2} \mp 1 \mp \frac{T_3 e^{-i2h_m a_1}}{T_1+T_2}$$

$$T_{1m} = \cos g_m(a_2-a_1),\ T_{2m} = \frac{i}{2}\left[t_m + \frac{1}{t_m}\right]\sin g_m(a_2-a_1)$$
$$T_{3m} = \frac{i}{2}\left[t_m - \frac{1}{t_m}\right]\sin g_m(a_2-a_1),\ t = \frac{g_m\varepsilon_{r1}}{h_m\varepsilon_{r2}}$$

We assume normal incidence ($\theta = 0$), and further that $kb\sqrt{\varepsilon_r} < \frac{3\pi}{2}$; the latter condition ensures that only the $m = 0$, 1 and 2 modes of the parallel plate waveguide will be above cutoff and all other modes will be evanescent. Retaining these modes terms in (2) ($m = 0, 1$ and 2), we get a degenerate-kernel integral equation whose solution can be found by well-known methods:

$$\tilde{x}^{\pm}(0) = \frac{2kb}{\pi\sqrt{\varepsilon_{r1}}}\left[\frac{N_0^{\pm}}{1+\frac{2h_0h_2b^2q^2N_0^{\pm}N_2^{\pm}}{\pi^6\varepsilon_{r1}^2}}\right] \quad (3)$$

where $N_0^{\pm} = \frac{\Gamma_0^{\pm}}{1-\frac{kb\Gamma_0^{\pm}}{2\sqrt{\varepsilon_{r1}}}I_{00}}$ and $N_2^{\pm} = \frac{\Gamma_2^{\pm}}{1-\frac{h_2b\Gamma_2^{\pm}}{\varepsilon_{r1}}I_{22}}$. For $kb \ll 1$, $I_{00} \simeq 2 - \frac{4i}{\pi}\ln\frac{\phi}{kb}$ where $\ln\phi = \frac{3}{2} - \gamma_E$, $\gamma_E \simeq 0.5772\ldots$ is Euler's constant and $I_{22} \simeq \frac{2i}{\pi^2}Si(2\pi) \simeq i0.2874\ldots$, Si being the sine integral function [7][8][9]. Now we can determine the magnitude of the transmission through the dielectric slot. As in [7], we define the (amplitude) transmission factor as:

$$TF = \frac{\int_{-b}^{b} E_y(y,-a)\,dy}{\int_{-b}^{b}|E^{inc}(y,0)|\,dy} = \frac{1}{\sqrt{\varepsilon_{r1}}}\left[\frac{N_0^+}{1+\frac{2h_0h_2b^2q^2N_0^+N_2^+}{\pi^6\varepsilon_{r1}^2}} - \frac{N_0^-}{1+\frac{2h_0h_2b^2q^2N_0^-N_2^-}{\pi^6\varepsilon_{r1}^2}}\right] \quad (4)$$

having used (3) to obtain the final formula.

III. Results

Previously [9], we studied the effect of introducing the gap within the slot on broad TEM mode ($m = 0$) resonances, and showed that the gap size controls the frequency intervals between TEM resonances. In [8] we showed that including higher order modes reveals new sharp Fano resonances. In the present study, we will investigate the effect of introducing the gap within the slot on higher order mode resonances, for parameter values $a_3 = 2$ mm, $2b = 4$ mm, $a_2 = 1.3$ mm,

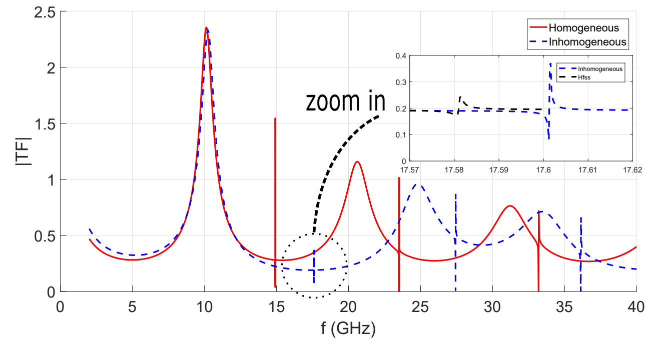

Fig. 2. Transmission Factor TF of homogeneous and inhomogeneous symmetrically filled ($a_3 - a_2 = a_1$) slots: $a_3 = 2$ mm, $b = 2$ mm, $\varepsilon_{r1} = 50$, $\varepsilon_{r2} = 20$ and gap=0.6 mm. Inset shows comparison with HFSS results.

$\varepsilon_{r1} = 50$ and $\varepsilon_{r2} = 20$. A comparison among results of the inhomogeneous gap formula (4), the homogeneous slot (no gap) formula from [8] and a full wave HFSS simulation is shown in Figure 2. The results show that the higher order resonances locations and magnitudes will be determined by the gap's size and ε_{r2}. With $\varepsilon_{r2} < \varepsilon_{r1}$, increasing the size of gap shifts the higher order mode resonances to higher frequencies and decreases their magnitudes. If we choose ε_{r2} such that the higher-order ($m = 2$) mode is propagating, the magntude reduction is moderate. On the other hand, if we choose ε_{r2} such that the higher order mode is evanescent, the magnitude reduction is far more dramatic.

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