Physical bounds on the MIMO capacity for small antennas

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Antenna current optimization can be used to determined physical bounds for antennas of arbitrary shape (Gustafsson *et al.* 2015). The performance can be quantified in *e.g.*, the Q-factor, gain, directivity, and efficiency. MIMO (multiple input multiple output) antennas are more intricate and it is in general not sufficient with a single quantity to determine their performance. Here, a convex optimization problem for the maximum capacity with fixed Q-factor and signal-to-noise ratio is formulated in the covariance matrix of the current distribution. This problem is simplified by a reduction of the number of degrees of freedom using current modes.

A classical MIMO system is modeled as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{x}$ is the $N \times 1$ matrix with input signals, $\mathbf{y}$ the $M \times 1$ matrix with output signals, $\mathbf{n}$ the $M \times 1$ matrix with additive noise, and $\mathbf{H}$ the $M \times N$ channel matrix (Paulraj *et al.* 2003). The channel matrix models the transmitting and receiving antennas and the wave propagation between the antennas.

To determine physical bounds on MIMO antennas, the channel matrix $\mathbf{H}$ should model the channel between an arbitrary antenna and an idealized receiver. The transmitting antenna is hence modeled with its current distribution using a MoM (method-of-moments) approximation where each basis function corresponds to an element of $\mathbf{x}$. The receiver is modeled with the radiated spherical modes such that each mode is an element in $\mathbf{y}$ (Gustafsson and Nordebo 2007). This leads to a MIMO system of infinite dimension as N increases with the mesh refinement and M with the number of included spherical modes, and therefore an unlimited capacity. This unrealistic unbounded capacity has an analog in superdirectivity and can be eliminated by constraints on the efficiency or Q-factor.

The efficiency and Q-factor are expressed as quadratic forms in the current density, where the stored energy in (Vandenbosch 2010) is used. Moreover, the Q-factor is determined from the quotient between the average stored energy and average dissipated power. This leads to a convex optimization problem that maximizes the ergodic capacity for a fixed signal-to-noise ratio and Q-factor. The convex optimization problem is a semi-definite program expressed in the covariance matrix of the current distribution which has $(N^2 + N)/2$ parameters and if is hence difficult to solve the problem for a fine mesh (large N). A model order reduction approach using characteristic and energy modes are used to reduce the dimension of the optimization problem.

Numerical results are used to illustrate the results. The solution of the optimization problem provides the maximum capacity and the covariance matrix of the current distribution. The capacity and covariance matrix are compared with the minimum Q current modes on the antenna structure (Capek *et al.* 2016).