An Element-Diagonal Preconditioner for the Volume Electric-Field Integral Equation

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For bioelectromagnetic (BioEM) problems in the UHF band, the condition number of the method of moments (MoM) system increases with the frequency, resulting in a larger number of iterations required for convergence ($\propto$ solve cost) (J. W. Massey, M.S. Thesis, The University of Texas at Austin, 2015). Preconditioners have been applied to improve the conditioning of the system, such as the iterative near-field preconditioner or sparse-approximate-inverse preconditioner (T. Malas and L. Gürel, *SIAM J. Sci. Comput.*, 2009), but their benefits must be balanced with the added complexity of filling them and applying them at each iteration.

An element-diagonal preconditioner proposed in (J. Aronsson, M. Shafieipour, and V. Okhmatovski in *28th Annu. Review Progress Applied Comput. Electromagn.*, 2012) for RWG basis functions for the MoM solution of the combined-field integral equation (CFIE) is extended and applied to the volume electric-field integral equation (V-EFIE). The element-diagonal preconditioner for the V-EFIE is formed by filling the sparse matrix $\mathbf{P}_{\mathrm{ED}}$ whose non-zero entries are located at $[i, j]$ where the basis/testing functions $\mathbf{f}_i$ and $\mathbf{f}_j$ have at least one overlapping patch. Using this "near-field" definition, the number of non-zero entries is $O(N)$, specifically $< 7N$ for SWG basis functions and $< 11N$ for volumetric rooftop basis functions, where N is the number of unknowns in the MoM system.

For the original MoM system, $\mathbf{ZI} = \mathbf{V}$, a left-preconditioner, $\mathbf{P}_{\mathrm{L}}^{-\mathbf{1}}$, can be applied as $\mathbf{P}_{\mathrm{L}}^{-\mathbf{1}}\mathbf{ZI} = \mathbf{P}_{\mathrm{L}}^{-\mathbf{1}}\mathbf{V}$ where $\mathbf{P}_{\mathrm{L}}^{-\mathbf{1}}$ is designed to reduce the spectrum of $\mathbf{Z}$ and can be calculated within a practical time frame. The element-diagonal preconditioner matrix can be inverted using a sparse inverter, e.g. SuperLU, and is then applied with an additional matrix-vector multiplication ($\mathbf{P}_{\mathrm{L}}^{-\mathbf{1}}(\mathbf{ZI})$). Alternatively, a right-preconditioner, $\mathbf{P}_{\mathrm{R}}$ can be applied in a forward-implicit manner where $(\mathbf{ZP}_{\mathrm{R}}^{-\mathbf{1}})(\mathbf{P}_{\mathrm{R}}\mathbf{I}) = \mathbf{V}$. The flexible generalized minimal residual (FGMRES) solver in PETSc provides an inner-outer framework that iteratively inverts the preconditioner for each outer MoM system iteration, circumventing the overhead of directly inverting $\mathbf{P}_{\mathrm{R}}$.

In this article, the impedance matrix-vector product, $\mathbf{ZI}$, is accelerated using the adaptive integral method (AIM) (F. Wei and A E. Yılmaz, *IEEE Trans. Antennas Propag.*, 2014). Results will be shown for problems in the Austin Benchmark Suite for Computational Bioelectromagnetics comparing the MoM system performance with a diagonal preconditioner ($\mathbf{P}_{\mathrm{L}} = \mathrm{diag}(\mathbf{Z})$) to the element-diagonal preconditioner implemented using either a sparse-direct inverse solution ($\mathbf{P}_{\mathrm{L}} = \mathbf{P}_{\mathrm{ED}}$) or as a right-preconditioner ($\mathbf{P}_{\mathrm{R}} = \mathbf{P}_{\mathrm{ED}}$) in an inner-outer framework for the FGMRES solver.