Monte Carlo approach for investigating the fabrications imperfections for metasurface-lenses

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Abstract—**In this paper, we introduce and evaluate, for metasurfaces, parameters such as the intercept factor and the slope error usually defined for solar concentrators in the realm of ray-optics. After proposing definitions valid in physical optics, we put forward an approach to calculate them. As examples, we design three different lenses based on three specific unit cells and assess them numerically. The concept allows for the comparison of the efficiency of the metasurfaces, their sensitivities to fabrication imperfections and will be critical for practical systems.**

Keywords— Metasurface; Thin films; Concentrators; Lenses.

I. INTRODUCTION

Metasurfaces, as a two-dimensional version of metamaterials, have raised significant attention due to the simplified design afforded by generalized Snell's laws of reflection and refraction [1]. They consist of arrangements of subwavelength elements and provide powerful solutions to control the phase, the amplitude and the polarization of waves at subwavelength scales. The metasurfaces can be theoretically modeled in terms of surface polarizabilities (electric and magnetic) with physical bounds [2]. They offer a promising platform for applications including optical devices for beam splitters [3], carpet cloaking [4] and lenses [5-7]. Among these applications, metasurface metalens and concentrators are receiving considerable attention due to their capabilities for flat and integrable optics, super-focusing, super-imaging and solar energy.

Conventional lenses are bulky as they rely on the Snell-Descartes laws of refraction and propagation over large distances—compared to the wavelength—to focus light. On the other hand, metalenses can concentrate light with very thin surfaces—of the order of micrometers— by imposing an abrupt phase-shift to light at some interface. For instance, a parabolic metallic concentrator can be replaced by a thin and flat metasurface which provides, to a normally incoming plane wave, the parabolic phase-shift given by:

$$\Phi = k_0(\sqrt{x^2 + f^2} - f) \qquad \textbf{(1)}$$

where k_0 is the free space wave-vector, x is the distance between the considered element and the center of the lens and f is the focal length. In general, only the focusing efficiency is considered to determine the quality of metalenses [6-7]. The latter corresponds to the ratio of the power incident on the focus to the power incident on the lens. In the solar concentrator field, the efficiency is defined as the ratio of solar energy collected by the receiver—an optical absorber—to that intercepted by the lenses. The total optical efficiency of a solar concentrator is given by the combination of the so-called intercept factor, the reflectance of the concentrators, and the absorbance of the latter [9]. Since the efficiency of an energy concentrator is extremely sensitive to its geometrical parameters, it is essential to develop methods that allow their optimization.

In this paper, we introduce a method to compare the quality of concentrators in the realm of metasurfaces. Specifically, we generalize the concepts of the slope error and the intercept factor. An approach based on finite difference time domain (FDTD) simulations is proposed to evaluate the efficiency of concentrators. As examples, we design in the optical domain three metasurfaces based on different unit cells (with cylindrical, rectangular and ellipsoidal elements) made of titanium dioxide (TiO_2) [8-10]. We compare the three designs with our approach and show that the rectangular element has the minimal sensitivity to fabrication imperfections.

II. INTERCEPT FACTOR AND SLOPE ERROR

In the solar concentrator field [11-15], the intercept factor and the slope error allow the description of the imperfections of a solar concentrator. For traditional solar concentrators, the curved mirror has been used to bend light and focus it, which lead to the definition of the slope error. Metasurfaces are generally flat and rely on phase gradients and interferences to focus light. Hence, we define the intercept factor as the ratio of the integrated power on the receiver to the power incident on the metasurface. The corresponding equations for the ideal and real cases are respectively:

$$\begin{cases} \sin\theta_i - \sin\theta = \frac{1}{k_0}\frac{d\varphi(x)}{dx} \text{ for the ideal case} \\ \sin\theta_i - \sin(\theta + \delta\theta) = \frac{1}{k_0}\frac{d(\varphi(x)+\delta\varphi(x))}{dx} \text{ for the real case} \end{cases} \qquad \textbf{(2)}$$

where θ_i is the angle of the incident plane wave, θ the angle of the reflected wave in the ideal case and $\theta + \delta\theta$ the angle of the reflected wave in the real case. This leads us to define the equivalent of the slope error, a unitless phase gradient error.

$$Slope\ Error = \left|\frac{1}{k_0}\frac{d\delta\varphi}{dx}\right| \qquad \textbf{(3)}$$

In this case, $d\delta\varphi/dx$ can be approximated as the ratio of the phase difference between two adjacent elements to dx, the distance between two resonators.

III. QUANTITATIVE ANALYSIS OF INTERCEPT FACTOR AND SLOPE ERROR IN NON-PERFECT METASURFACE

In order to show how such fabrication imperfections can degrade the efficiency of a metasurface, and how they can be characterized by the intercept factor and by the slope error, we designed metasurfaces with three different geometrical structures: cylinders (Fig. 1(a)), rectangular parallelepipeds (Fig. 1(b)), and ellipses (Fig. 1(c)) that are widely used to design elements of metasurfaces [5-7]. Most of fabrication imperfections result in difference between the fabricated and the ideal sizes of the resonant element. In real experiments, such value is around 10 nm for conventional electron beam lithography techniques [16]. We modeled the fabrication imperfections as a random phase noise that adds to the phase shift of the elements. Hence, the total phase at each element is given by, $\Phi_{real} = \Phi + \varepsilon\ \Delta P(\Phi)$. Where Φ_{real} is the phase of the element with fabrication imperfections, Φ is the ideal parabolic phase. ε is a random number between -0.5 and 0.5, picked up from a uniform distribution. ΔP is the magnitude of the random number which is a function of the phase shift Φ. To statistically analyze our metasurface lenses, we run 100 simulations (Monte Carlo approach) for each element using a homemade FTTD code. Each simulation was given a certain magnitude of the noise related to the type fabrication imperfections. Fig. 1(d) shows the intercept factor as a function of fabrication imperfections. For a structure without fabrication imperfections, the value of the intercept factor is equal to unity. For example, for a fabrication imperfection value equal to 10 nm for cylinders and ellipses, the value of the intercept factor is about 0.72. In the case of the rectangular parallelepipeds a similar intercept factor of 0.71 is obtained for twice as larger fabrication imperfections (20nm). For this same value of fabrication imperfections in the case of a cylinder and the ellipse, the value of the intercept factor is 3 times smaller than the rectangular one. Fig. 1(e) presents the slope error as a function of the fabrication imperfections. The ellipses and the cylinders metasurface have the approximately equal slope error value that is about twice that of the rectangle. This proves that the rectangular parallelepipeds are less sensitive to the considered fabrication imperfections, and they would be more advantageous to use to design highly efficient metasurfaces.

In conclusion, we presented an approach to evaluate the robustness of metasurface lenses to fabrication imperfections. We started by describing the general methods used, and investigated three different geometries as unit cell elements with cylinders, rectangular parallelepipeds, and ellipses cross sections. We studied the effects of imperfection via the intercept factor and the slope error. Our approach can provide a guidance to design large scale and highly efficiency metasurface concentrators.

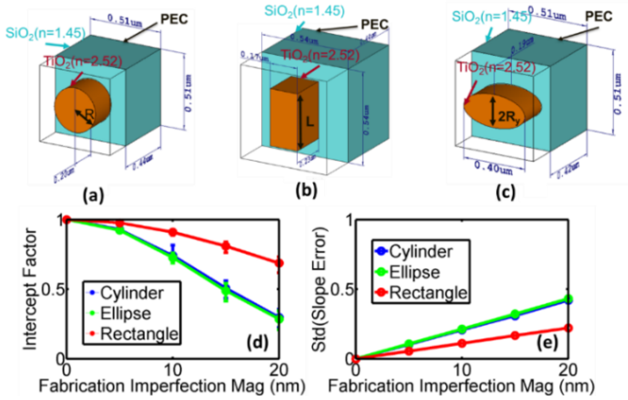

Fig.1. (a-c) Schematic of a conventional metasurface structure. (Cylinders, rectangular parallelepipeds and ellipses.) (d) Intercept factor (e) Slope error

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