Extremely Low-Frequency Electromagnetic Computation by Using the Mixed Spectral-Element Method to Overcome the Low-Frequency Breakdown

Yuanguo Zhou * (1), Mingwei Zhuang(1), Na Liu(1) and Qing Huo Liu* (2), *Fellow, IEEE*
(1) Institute of Electromagnetics and Acoustics & Department of Electronic Science, Xiamen University, Xiamen 361005, China
(2) Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA

For subsurface electromagnetic exploration problems, the low-frequency EM pulses are employed to penetrate the lossy ground and obtain deep geologic information. However, the challenge in low-frequency electromagnetic subsurface numerical computation lies in the well-known low-frequency breakdown phenomenon, which leads to the singularity of the system matrix. One reason for this is that the Gauss' law is not enforced explicitly. Such a low-frequency breakdown problem makes it difficult to solve a large system by an iterative method because of slow convergence, and prevents even the direct solution when frequency is below 0.1 Hz. As a result, significant attention has been paid to the low-frequency breakdown problem in both integral equations and partial differential equations. Conventional methods is to switch basic functions to overcome the low-frequency breakdown problem, but this method only partly removes the breakdown problem since at extremely low-frequencies the system matrix equation is still ill-conditioned, leading to slowly converging iterative solutions. More recent approaches focus on better selections of basis functions and Helmholtz decomposition.

In this work, we propose a new mixed spectral element method (mixed SEM) by applying the divergence-free equation in Kikuchi's scheme to eliminate the low-frequency breakdown problem, and apply this method to solve the complex 2.5-D and 3-D subsurface electromagnetic exploration problem. We employ the curl-conforming vector basis functions to expand the fields, but use linear nodal element basis functions to expand the Lagrange multiplier so that Gauss' law can be satisfied. Because the Gauss' law is considered, all physical laws are honored, leading to stable solution and making system matrix well-conditioned even at extremely low frequencies. This method can obtain accurate results with much less time and memory than commercial software due to the well-conditioned matrix and the use of the SEM.

One of our applications is the surface-to-borehole electromagnetic system which is the evolution of the controlled source electromagnetic technique, and surface to surface electromagnetic technique. The system has been a very important research project in reservoir exploration because of its unique measurement method. It uses low-frequency controlled sources located on the ground surface as a transmitter and a magnetometer in a borehole as the detector. In this work, our general computational technique based on the mixed SEM is used to overcome the low-frequency breakdown phenomenon, but apply it to study the response of the surface-to-borehole electromagnetic system in the frequency domain; this response can be further transformed into the time domain by Fourier transform if a transient response is needed. Several application examples will be demonstrated.