On invisible bodies, nonradiating sources, and embedded eigenstates

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The past two decades have witnessed large efforts toward applying engineered metamaterials to tailor the electromagnetic scattering of material bodies in anomalous and extreme ways. As the most notable example of these efforts, invisibility devices have been studied and fabricated for operation in different frequency ranges and for different types of waves. Very recently, new directions are emerging in this fascinating field, including new forms of non-scattering configurations, such as *anapolar scatterers* [e.g., A. E. Miroshnichenko, et al., Nat. Commun. **6**, 8069 (2015)], and scatterers supporting nonradiating modes, or *embedded eigenstates* [e.g., F. Monticone and A. Alù, Phys. Rev. Lett. **112**, 213903 (2014).]. Significant confusion, however, is accompanying these new developments, especially in relation to the excitation of nonradiating field configurations, and the role of reciprocity.

In our talk, we will start by reviewing the similarities and differences of various forms of cloaking devices, anapolar scatterers, structures supporting embedded eigenstates, etc. We will show that embedded eigenstates correspond to nonradiating *eigenmodes*, namely, self-sustained field distributions that do not radiate. Such trapped states can exist in isolated three-dimensional objects only if the material properties assume singular values [M. G. Silveirinha, Phys. Rev. A **89**, 23813 (2014)], and they determine scattering resonances with diverging Q factors, as a scattering zero moves arbitrarily close to a resonance peak. From first-principle considerations stemming from Lorentz reciprocity, we will show that it is exactly because of their eigenmodal nature that embedded eigenstates cannot be excited from external sources (although there is no limit on how close the nonradiating condition can be approached). Conversely, invisible bodies, such as cloaking devices and anapolar scatterers, support nonradiating field distributions that are *not* self-sustained eigenmodes, as they do not satisfy on their own the electromagnetic boundary conditions on the surface of the scatterer, without the presence of the excitation field. As a result, there is nothing surprising in the ability to externally excite these nonradiating field configurations, as they actually exist only in the presence of a suitable external excitation field. The implications of reciprocity can be clarified by performing time-domain numerical simulations in which scatterers supporting quasi-embedded eigenstates or anapolar field distributions are illuminated by a propagating wave that rapidly ends at a given time instant. In the case of a quasi-embedded eigenstate, the induced field distribution keeps oscillating within the scatterer after the excitation is terminated, slowly radiating energy in free-space with a decay time exactly equal – a direct consequence of reciprocity – to the time necessary to reach steady state (in the limit of an ideal embedded eigenstate, the decay time is infinite, i.e., the eigenmode is nonradiating). In the case of a non-scattering anapolar distribution, instead, the scatterer rapidly releases the stored energy in the form of radiation when the excitation ends, as the boundary conditions are no longer satisfied when the external incident field vanishes. This difference can also be appreciated by inspecting the poles of the scattering coefficients on the complex frequency plane, which are associated to the eigenmodes of the scatterer. An embedded eigenstate corresponds to a single scattering pole arbitrarily close to the real frequency axis, a result of the disappearance of radiation leakage. In the case of cloaked objects or anapolar scatterers, instead, the nonradiating field distribution is determined by the excitation of multiple eigenmodes of the scatterer corresponding to different poles at complex frequencies. These results and considerations will be further elaborated and discussed in our talk.

Our work sheds light on the general properties of nonradiating field configurations, elucidating relevant differences between the concepts of invisibility and embedded eigenstates, as well as the implications of Lorentz reciprocity for the external excitation of nonradiating modes.