New frequency domain discontinuous Galerkin method with domain decomposition technique for electromagnetic modeling

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Conventional frequency domain discontinuous Galerkin (DGFD) methods employ the Robin transmission conditions to couple different subdomains (S.-C. Lee, M. N. Vouvakis, and J.-F. Lee, J. Comput. Phys., 203, 1, 1-21, 2005). Peng and Lee introduce a second-order transmission boundary condition, which shows better convergence than Robin transmission conditions (Z. Peng and J.-F. Lee, J. Comput. Phys., 229, 16, 5615-5629, 2010) .

In this paper, we introduce a novel approach to evaluate the flux term with the Riemann solver (upwind flux) based on the second-order wave equation. The Riemann solver is widely employed for time domain DG methods based on the first-order Maxwell's curl equations, and it is demonstrated to have an optimal $O(h^{p+1})$ convergence rate (J. S. Hesthaven and T. Warburton, J. Comput. Phys., 181, 1, 186-221, 2002). Thus, to exploit the good convergence of the Riemann solver, we propose a new approach to incorporate it into frequency domain DG formulation. Specifically, the flux term in the weak form of the second-order wave equation is evaluated with one of the equation in the Riemann solver, which involves both **E** and **H** fields. To obtain a balanced system, the other equation in the Riemann solver is also tested, but only on the interfaces between subdomains. Finally a coupled linear system is ready for solution, which only involves the **E** unknowns plus surface **H** unknowns for each subdomain. In this DGFD method, domain decomposition technique is also employed to split the original computational region into multiple subdomains. In each subdomain different element types (e.g., tetrahedron and hexahedron) and orders of basis functions can be employed to take advantage of the model geometry and meanwhile to reduce the degrees of freedom. Moreover, non-conformal mesh is allowed between subdomains. For the resultant coupled linear system, efficient linear solvers are employed to accelerate the solution process. When all the subdomains are in a sequential order, the LDU algorithm is used as a efficient direct solver (Q. Sun, L. E. Tobon, Q. Ren, Y. Hu, and Q. H. Liu, IEEE Trans. Compon. Packag. Manuf. Technol., 5, 12, 1839-1849, 2015). For large scale problems, the DoFs can be too large to be solved by direct solvers. In this situation, iterative solvers, such as symmetric successive over-relaxation (SSOR) preconditioned BiCGStab method, are used. Numerical results demonstrate that the proposed DGFD method shows better convergence than Robin transmission conditions. The proposed DGFD method can facilitate multiscale modeling of electromagnetic problems in frequency domain.