Sparse Solution of Integral Equation Formulations of Multiple Scattering Problems in a Directional Plane Wave Basis

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Efficient, O(N) sparse direct solution methods have been developed for low-frequency electromagnetic applications. One class of such methods is based on the physical concept of spatial localization. Unfortunately, existing methods based on spatial localization quickly become computationally prohibitive as the frequency increases, scaling faster than $O(N^2)$ as the frequency increases.

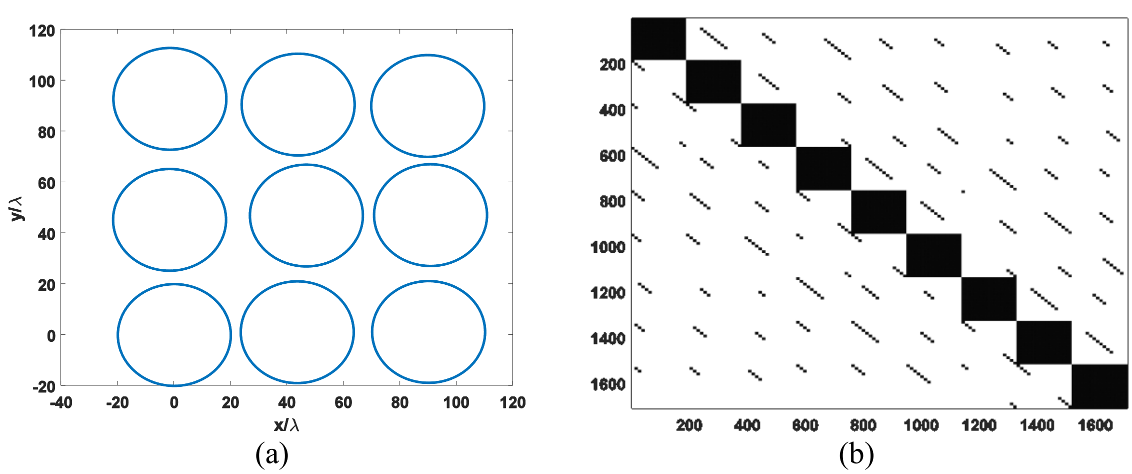
In this paper we report a method for sparsifying the system matrix based on the use of directional plane wave sources and receivers for the problem of scattering from a collection of perfectly conducting objects such as those indicated in Figure 1(a). The new solver proceeds by first extracting all localizing sources (see above) from the system matrix. The remaining, non-localizable sources are subsequently transformed using directional plane-wave basis functions. The directional basis functions are determined by solving an auxiliary problem for each spatial group (in the example shown below, each PEC cylinder is a spatial group). For TMz polarization, the required directional source vectors, J_z, are determined via discretization and subsequent algebraic manipulation of a plane wave transform of the form,

$$E(\phi) = \int_C J_z(\boldsymbol{\rho}) e^{-j\mathbf{k}(\phi)\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_c)} \, d\boldsymbol{\rho} \tag{1}$$

where the desired J_z radiate only into one of many angular directions. A similar analysis is performed to determine localizing receivers. If **X** and **Y** indicate the matrices containing the directional sources and receivers, then the transformed system matrix in the directional plane wave basis is

$$\mathbf{D} = \mathbf{Y}^T \mathbf{Z} \mathbf{X} \tag{2}$$

The structure of **D** is illustrated in Figure 1(b). Each off-diagonal block, corresponding to interactions between a pair of scatterers, is a diagonal sub-matrix. The self-interaction blocks along the diagonal are generally full. However, it will be shown during the presentation that use of a general, combined source integral equation renders the diagonal blocks nearly diagonal. This in turn enables certain sparse, direct solution strategies.



(a) (b)

Figure 1. TMz PEC scattering configuration (a) and corresponding directional plane wave transform of the system matrix (b) using a tolerance of 1E-6; all localized/non-radiating DOF have been removed from the problem.