# Efficient Parallelization of MLFMA with a Hybrid Global Interpolation/Anterpolation Scheme

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Parallelization of the multilevel fast multipole algorithm (MLFMA) has been the subject of much research over the past decade or so. As a result of this effort a number of methods have emerged for solving large-scale problems. These include the hierarchical partitioning (HiP) strategies (Ergul & Gurel, IEEE Trans. Antennas. Prop., 57.6, p. 1740, 2009; Michiels et al., IEEE Trans. Antennas. Prop., 63.2, pp. 796–799, 2015) and the local essential tree (LET) strategy (Melapudi et al., IEEE Trans. Antennas. Prop., 59.7, pp. 2565–2577, 2011), upon which this work is based. The ability to perform large-scale electromagnetic simulations with MLFMA is predicated on the ability to compute deep trees. While large problems have been solved with the HiP schemes, they suffer from two important problems; (i) the hierarchical strategy imposes a strict $2^n$ or $4^n$ requirement on the number of processes that may be used, and (ii) they employ *local* interpolation of radiation fields. While the halo exchange-like local interpolation algorithm is practical due to its low communication overhead, these operations are not *band-limited*. It is thus necessary to oversample the radiated fields by a factor of as much as 10 to obtain accuracy comparable to approaches based on global interpolation, significantly increasing the memory requirements per node.

In contrast, the LET strategy provides fine-grained error control using a minimal number of samples; however, the efficient vector spherical harmonic (VSH) transform employed here (B. Shanker, et al., IEEE Trans. Antennas. Prop., 51.3, pp. 628–641, 2003) requires the calculation and storage of an interpolation matrix whose memory cost scales as $\mathcal{O}\left(N_{\theta,l}^3\right)$, where $N_{\theta,l}$ represents the number of samples in the $\theta$ direction at level $l$. At the upper levels of a deep tree, this cost becomes prohibitive. Our remedy to this bottleneck is to choose a level above which we transition from the VSH representation to a plane-wave representation on a uniform grid to leverage fast Fourier transforms (FFTs) in both $\theta$ and $\phi$ directions (J. Sarvas, SIAM J. Numer. Anal, 41.6, pp. 2180–2196, 2003), eliminating the memory bottleneck at the cost of doubling the sampling rate in $\theta$. Additionally, the FFTs must be taken on the cartesian components of the radiated fields, incurring some overhead in mapping between spherical and cartesian vector bases.

The overall cost per node of interpolation/anterpolation grows as one ascends the tree. In the LET scheme, certain processes which straddle domain boundaries at the uppermost levels of the tree may be responsible for computing an inordinate number of interpolations/anterpolations at these levels. This imbalance has a deleterious effect on the efficiency of the algorithm. This may be alleviated by parallelizing the interpolations/anterpolations for nodes that appear in the subtree of multiple processes (duplicate nodes).

In this paper, we describe a scalable parallel algorithm for FFT-based operations on nodes shared by multiple processes at the upper levels of the tree to facilitate large-scale electromagnetic simulation. We will present convergence, load balance, and scaling results to demonstrate clearly the robustness of this algorithm.