Algorithms for Adaptive Trees Within a Parallel MLFMA

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Fast simulation methods for multiscale problems in electromagnetics (EM) has become the focus of much research in recent years. A large part of this effort has been devoted to extending the multilevel fast multipole algorithm (MLFMA) to low frequencies via partial-wave decomposition, approximate diagonalization of the translation function, or hybridization with Cartesian expansion of the kernel in low-frequency regimes. At the same time, further efforts have been poured into developing efficient parallelizations of the MLFMA, but these two paths have not intersected. The present work describes an efficient parallel MLFMA with an adaptive tree to maximize computational efficiency without sacrificing accuracy.

In the traditional MLFMA, the densest region of discretization dictates the leaf box size throughout the entire tree. The leaf box size must be chosen such that the near-field cost does not monopolize computational resources. For geometries with uniform discretization and spatial distribution, this does not present a problem; however, for multiscale geometries, the non-uniform spatial distribution of degrees of freedom (DoF) represents a computational bottleneck within the uniform-tree paradigm. Swaths of the uniform tree may be relatively sparsely populated, significantly hampering computational efficiency.

To avoid unnecessary tree operations within these regions, the sparse boxes should be merged so that each leaf box has a roughly equal number of points within. The resulting tree structure is nonuniform, i.e. the leaves of the tree may reside at any arbitrary level. However, unbounded merging of the tree presents two problems: (i) the number of possible near or far interactions for a given box in the tree may be extremely large in regions of high contrast discretization, and (ii) this necessitates interactions between boxes at different levels of the tree. Because the accuracy of the translation operator depends upon the ratio of the box size to the translation distance, interactions between boxes of vastly different size will be highly inaccurate. For these reasons, the tree must be merged so that the leaf box sizes do not change rapidly within a region of space. This is achieved by constraining the merging procedure with the 2:1 balance constraint (H. Sundar, et al., SIAM J. Sci. Comput., 30, 5, 2675–2708, 2008).

In this work, we describe an adaptive parallel MLFMA and present a different parallel algorithm for constructing and computing on the adaptive tree subject to the 2:1 balance constraint. We define the necessary operators for interactions between well-separated boxes at adjacent levels and discuss related accuracy considerations. We also present an efficient method for computing these non-diagonal interactions and an automatic load balancing mechanism. We demonstrate the effectiveness of our approach through memory profiling, timings, and a strong scaling study.