An $\mathcal{H}$-Matrix Accelerated Solution of a New Single-Source Integral Equation for Scattering on Penetrable Objects

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Recently, a novel single-source integral equation (SSIE) has been proposed for 2D (A. Menshov, et.al., IEEE T-MTT, no. 1, vol. 61, pp. 341–350, 2013) and 3D (F. Sheikh, et.al., submitted to IEEE T-APS, 2016) scattering problems on homogeneous dielectric objects. Unlike the previously known SSIEs, the new equation involves only one product of integral operators and requires only electric-field-type of Green's function while still reducing the number of unknowns by half compared to traditional surface integral equation formulations. However, memory requirements and computational complexity of dense matrix operations and storage for naive MoM solution of the novel SSIE limit its application to small problems. In this work, we investigate a fast direct and iterative solutions for the new SSIE based on the hierarchical matrix ($\mathcal{H}$ -matrix) framework (W. Hackbusch, Computing, 62, 89–108, 1999).

At first, $\mathcal{H}$-matrix based solver hierarchically divides the structure of interest into sub-domains such that the matrix is decomposed into sub-blocks each representing the interaction between a pair of sub-domains. Then, for each well-separated pair of sub-domains τ and σ, the corresponding block $Z_{\tau\times\sigma}$ is expected to be low rank and is represented in the $Z_{\tau\times\sigma} \approx AB^H$ form. The accuracy of the approximation is dependent on the low-rank factorization strategy, such as singular value decomposition, adaptive cross approximation, and grid-based algorithms. For the new SSIE, each of the entering integral operators (surface-to-surface, surface-to-volume, and volume-to-surface) is approximated using $\mathcal{H}$-matrices, and their combination is performed in the already compressed form. Acceleration of the MoM solution by $\mathcal{H}$-matrices results in the significant reduction of storage requirements and asymptotic complexity of matrix-matrix products, LU-decomposition, and back-subsitution.

At the conference, the performance of the algorithm in terms of the accuracy, computational time, and memory will be discussed and finally the optimized method will be applied to accelerate the solution of the new SSIE for the large-scale 3D scattering problem.