Fundamental limitations on the quality factor and related problems for small antennas

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In the last decade, the analysis of fundamental limits on antennas has advanced from canonical geometries (Chu 1948) to arbitrarily shaped radiators (Gustafsson *et al.* 2016). Certain fundamental bounds can be determined in the electrically small limit or be reformulated as convex optimization problems, however non-convex problems, such as unconstrained minimization of Q or total Q/G for larger structures, have remained unsolved.

In (Capek *et al.* 2016, arXiv:1612.07676), the problem of minimizing the quality factor Q for arbitrarily shaped radiators is reformulated in convex form using relaxation into a concave dual problem which is solved by a sequence of generalized eigenvalue problems. Applying modal decomposition and associated results from group theory (Schab and Bernhard 2016), it is further shown that the solution of the dual problem is the minimum quality factor Q, even in cases where inherent symmetries of the radiator appear to induce a duality gap. The proposed minimization procedure is computationally inexpensive and easy to implement, and the obtained numerical results are also compatible with published results based on modal currents (Jelinek and Capek 2016), convex optimization (Gustafsson and Nordebo 2013), and quasi-static approximations (Yaghjian *et al.* 2013).

The procedure to minimize the quality factor Q in (Capek *et al.* 2016) can directly be extended towards the maximization of the G/Q ratio and the analysis of radiators in the presence of regions with uncontrollable currents. Furthermore, the optimization procedure is generalized to minimize quality factor Q with respect to the arbitrarily weighted radiation patterns, or with prescribed magnitude of the electric and magnetic near-fields. The procedure is valid for volumetric and magnetic current densities as well. Using the proposed methodology, the class of solvable problems is significantly extended.

In this presentation, we review the results in (Capek *et al.* 2016) and present new results for multi-criteria optimization for lossy structures as well as multi-mode antennas.