On Higher Order Imperative in Computational Electromagnetics Through Benchmarking of Boundary Element Methods for Canonical Scattering Problems

M. Shafieipour[1], J. Aronsson[2], and V. Okhmatovski[3]

[1] Manitoba HVDC Center, Winnipeg, MB, Canada
[2] CEMWorks Inc., Winnipeg, MB, Canada
[3] University of Manitoba, Dept. of ECE, Winnipeg, MB, Canada

Methods of computational electromagnetics (CEM) such as Finite-Difference-Time-Domain (FDTD), Finite-Element-Method (FEM), Method of Moments (MoM), and others are widely used for design of today's wireless systems, analysis of biological effects of electromagnetic (EM) fields, prospecting for natural resources, and in various other important areas. While in most cases each of the above methods can be used for solution of a particular problem, the choice of a specific discretization approach is generally stipulated by a variety of factors. These factors include the level of required accuracy in approximation of EM fields, electrical size of the model, its material properties, and type of computational resources available for solution of the problem, among others. It can be shown that solution of the same problem can be achieved exponentially faster when the best method is used as opposed to its alternatives.

In this work we demonstrate such exponentially higher efficiency in solution of the scattering problems using the higher order (HO) boundary element methods (BEM) compared to their lower order (LO) counterparts. Specifically, we compare computational resources required for their solution with Rao-Wilton-Glisson (RWG) Method of Moments (*S. Rao, et.al., IEEE TAP, 30, pp. 409-418, 1982*) exhibiting $O(h)$ error behaviour, h being characteristic length of the surface discretization of the object as opposed to the HO Locally Corrected Nystrom (LCN) solutions (*L.F. Canino, et.al., JCP, 146, 2, pp. 627-663, 1998*) exhibiting $O(h^p)$ error behaviour, p being the order of LCN discretization. For problem of scattering on 33 wavelength diameter metal sphere achieving 2, 4, and 6 digits of precision with RWG MoM, requires 3 million, 4 billion, and 7 trillion unknowns, respectively. HO LCN method achieves such solutions with less than 0.5 million unknowns with orders 11, 13, and 15. This demonstrates the essence of the *Higher-Order Imperative* formulated by S. Wandzura in the form of the following three conclusions:

1. *Equating the problem "size" to the number of unknowns is incorrect*;
2. *Estimates of computational complexity without a notion of accuracy are meaningless*;
3. *HO algorithms are exponentially more efficient than LO counterparts.*

Various examples supporting these conclusions are shown in this work.