Numerical Analysis of Convergence of Sommerfeld Integrals in Layered Media Green's Functions

K. C. Durbhakula*[1], D. S. Kiddle[1], D. Chatterjee[1], A. M. Hassan[1], and M. S. Kluskens[2]

[1] CSEE Department, University of Missouri-Kansas City (UMKC), MO 64110, USA

[2] Radar Division, Naval Research Laboratory, Washington, DC 20735-5336

Sommerfeld integrals are ubiquitous in multilayer Green's functions problems, and their computational strategies have recently been comprehensively reviewed (K. A. Michalski and J. R. Mosig, *JEMWA*, pp. 281–317, Mar. 2016). A practical topology is shown in Fig. 1.

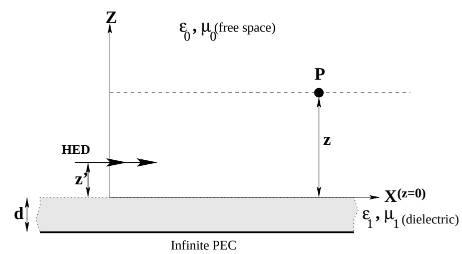

Figure 1: Hertzian electric dipole (HED) above a PEC-backed dielectric substrate; $l = 0$ and 1 in (1) below correspond to observation point $\mathbf{P}$ located in air and substrate regions, respectively.

The scalar z-component of the Green's function, for the geometry in Fig. 1, is given by:

$$\mathrm{G}_{zx}^{l=0,1} = \frac{(-1)^{l+1}\cos\phi}{2\pi k_0^2}\int_0^{+\infty}\Lambda_l(\xi)\{\xi^2\mathrm{J}_1(\xi\rho)\}d\xi. \tag{1}$$

In (1), the function $\Lambda_l(\xi)$ takes *different* forms as the receiver $\mathbf{P}$ moves from one ($l = 0$) media to the other ($l = 1$). The zeros of the denominator of $\Lambda_l(\xi)$ correspond to poles, and needs to be accounted for when evaluating the (improper) Sommerfeld integral in (1). The *weighted average*(WA) (Michalski and Mosig, *op. cit.*), and the *singularity subtraction*(SS) (E. Simsek, Q. H. Liu and B. Wei, *IEEE Trans. MTT*, pp. 216–225, Jan 2006) methods are two very distinct approaches in evaluating Sommerfeld integrals.

The number of proper surface wave poles contributing to evaluation of (1) increases with substrate electrical thickness $\frac{d}{\lambda}$. For multilayer geometries the algebraic form for $\Lambda_l(\xi)$ also substantially complicates the process. However, it is possible to accelerate both techniques by contour deformation, to avoid pole contributions. This approach will be gainfully utilized to examine the convergence of the WA and SS methods for single and double layer topologies.

The convergence results, and the associated analytical techniques for accelerating WA and SS methods, will be useful for developing full-wave solutions for understanding EMI/EMC and signal integrity issues in high-speed interconnect problems.