Diagonal Localization-based Direct Solver for Volume Integral Equations and Capacitive Extraction

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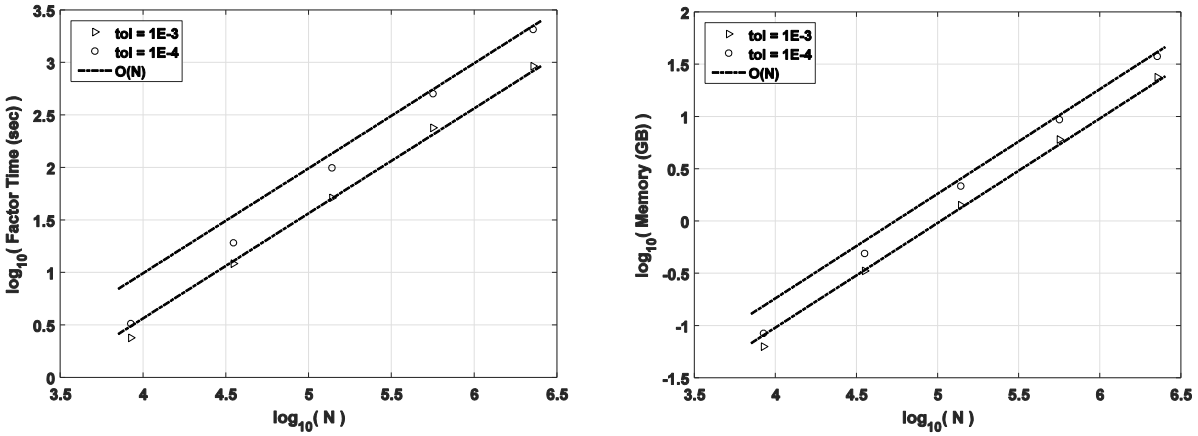
A sparse, localization-based direct solver for well-conditioned integral equation formulations was previously reported in (X. Xu and R.J. Adams, IEEE Trans. Antennas & Propag., 60, 1414-1424, 2012). The method uses localizing source transformations in conjunction with a unitary projection matrix to develop an upper-triangular block factorization. The method provides the first localization-based direct solver to provide an O(N) factorization cost for well-conditioned system matrices at low frequencies. This is accomplished using localizing sources with overlapping support. The degree of overlap is controlled using a shifted grid, wich enables all seams in the underlying oct-tree to be covered every three levels of the factorization.

In this paper, the O(N) overlapped, localizing factorization described above is diagonalized using the procedure reported for two-dimensional problems in (R.J. Adams and J.C. Young, Proc. ICEAA, 812-815, 2016). The diagonal factorization requires both localizing sources and localizing receivers, whereas the factorization algorithm reported above requires only localizing sources. As a result, the diagonal factorization increases the time required to perform the factorization somewhat due to the need to compute both source and field localization vectors; the O(N) complexity scaling of the factorization time is maintained (see Fig 1 below). The primary benefit of the diagonal factorization is the resulting memory savings. In particular, the constant in the O(N) memory scaling (see Fig 1 below) is reduced significantly. This enables the solution of larger problems for a given problem size. The diagonal structure of the factorization also reduces the time required to perform a solution for a given excitation. During this presentation, factorization implementation and performance will be discussed for a volume IE formulation of a magnetostatic problem and surface IE formulations of capacitive extraction problems.



(a) (b)

Figure 1. Factorization time (a) and memory usage (b) for localization-based direct solution for the magnetization induced in spherical shells of increasing radii with relative permeability μr = 200. Results are shown for input tolerances 1E-3 and 1E-4; in all cases, the relative RMS error in the factored representation is less than the indicated tolerance. Dashed lines show O(N) scaling.