Numerical Green's Function Based Augmented Electric Field Integral Equation for Inhomogeneous Media

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Electric field integral equation (EFIE) has been widely used in analyzing microwave circuits. One challenging issue for EFIE is that its impedance matrix becomes ill-conditioned when the frequency is low or the mesh unit size is small because the vector potential and scalar potential terms are not well balanced. In order to overcome this problem, augmented electric field integral equation (A-EFIE) has been developed where the current continuity equation is enforced additionally. As a result, an additional charge term is introduced into the equation which balances the vector potential and scalar potential terms at low-frequency. The Green's function in A-EFIE has an analytic form for piecewise-homogeneous media.

In the paper, A-EFIE is extended to analyze inhomogeneous media with numerical Green's function due to the lack of closed-form Green's function. It has been shown that the A-EFIE can actually be derived from the $\mathbf{A}$-ϕ formulations which are immune to the low-frequency breakdown. We start with the $\mathbf{A}$-ϕ formulations with generalized Lorenz gauge which is the governing equation in the dielectric region

$$\nabla \cdot \varepsilon_r \nabla \phi + \frac{\chi}{\varepsilon_0} \omega^2 \phi = -\rho_s / \varepsilon_0 \tag{1}$$

$$-\nabla \times \mu_r^{-1} \nabla \times \mathbf{A} + k_0^2 \varepsilon_r \mathbf{A} + \frac{\varepsilon_r}{c_0^2} \nabla \frac{\varepsilon_0}{\chi} \nabla \cdot \varepsilon_r \mathbf{A} = -\mu_0 \mathbf{J}_s \tag{2}$$

The numerical Green's function for charge $g_2(\mathbf{r}', \mathbf{r})$ and Green's function for electric current $\overline{\mathbf{G}}_2(\mathbf{r}', \mathbf{r})$ are obtained by solving $\mathbf{A}$ and ϕ equations separately with finite element method. Therefore, the numerical Green's function can be pre-calculated and reused. Since the scheme is independent of boundary condition, complex valued impedance boundary conditions are applied in order to eliminate the internal resonance problem in the dielectric region. With the solved inhomogeneous numerical Green's functions, two surface integral equations can be established for the dielectric region and free-space region, which are written as

$$-\mathbf{E}_{inc}/\eta_0 = \int_S d\mathbf{r}'\, g_1(\mathbf{r}, \mathbf{r}') \tilde{\mathbf{J}}_s(\mathbf{r}') + \int_S d\mathbf{r}'\, \nabla g_1(\mathbf{r}, \mathbf{r}') \tilde{\rho}_s(\mathbf{r}') - \int_S d\mathbf{r}'\, \nabla \times g_1(\mathbf{r}, \mathbf{r}') \cdot \tilde{\mathbf{M}}_s(\mathbf{r}') \tag{3}$$

$$0 = \int_S dS' \left[\tilde{\mathbf{J}}(\mathbf{r}') \cdot \overline{\mathbf{G}}_2(\mathbf{r}', \mathbf{r}) - \tilde{\rho}_s \nabla g_2(\mathbf{r}', \mathbf{r}) - \tilde{\mathbf{M}}_s(\mathbf{r}') \cdot \mu_r^{-1} \nabla' \times \overline{\mathbf{G}}_2(\mathbf{r}', \mathbf{r}) \right] \tag{4}$$

where $g_1(\mathbf{r}, \mathbf{r}')$ is the free-space Green's function and $\tilde{\mathbf{J}}_s = ik_0 \mathbf{J}_s$, $\tilde{\rho}_s = c_0 \rho_s$, $\tilde{\mathbf{M}}_s = \mathbf{M}_s / \eta_0$.

Since the magnetic current resides in the dual space of electric current, we use Rao-Wilton-Glisson (RWG) basis function to expand electric current and Buffa-Christiansen (BC) basis function to expand magnetic current. Then the total equation is tested with RWG basis function.

Numerical examples will be provided to demonstrate this numerical Green's function based A-EFIE is able to handle inhomogeneous dielectric problems.