Physical and Spectral Properties of the Electrostatic Nullspace in the Augmented Integral Formulations

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In order to model and simulate problems with compact sizes, fine structures, and especially multiscale dimensions, a low-frequency stable method is usually required to achieve a broadband computational regime. Integral equations are of the most intensively used numerical methods in solving electromagnetic problems. The magnetic field integral equation (MFIE) is known to be low-frequency stable due to its diagonal-dominant property in its matrix representation. An exceptional case is when MFIE is applied on the toroidal surfaces. Near-zero singular values are found to occur at the static limit, corresponding to the magnetostatic nullspace. The dimension of the magnetostatic nullspace equals to the number of the genus of the toroidal structure. The nullspace currents are presented in the form of global loops. This is the nullspace generated by the magnetic dipole.

On the other hand, the electrical field integral equation (EFIE) suffers from low-frequency breakdown problem. As a remedy, the augmented EFIE (A-EFIE) was proposed by adding the current continuity condition and using the charge as an extra unknown. By separating the vector potential and scalar potential apart, the A-EFIE method achieves good stability at low frequencies. However, at very low frequencies, the charge neutrality issue causes rank deficiency. The near-zero singular values make the condition number of the matrix to be very large. The corresponding nullspace bases are pure charge bases. This also happens in the potential-based $\mathbf{A}-\Phi$ formulation which uses the weak form the current continuity condition as the second equation. Mathematically, the charge neutrality condition is automatically satisfied due to the current continuity condition $\nabla\cdot\mathbf{J}=i\omega\rho$ and $\int_S \nabla\cdot\mathbf{J}=0$ for a single object. However, at very low frequencies, the charge neutrality condition may be violated since ω is approaching to zero. This brings in a nullspace and the dimension of this nullspace equals to the number of connected structures. Physically speaking, this nullspace is the electrostatic nullspace generated by electrical monopoles, which can be viewed as the duality case of the magnetostatic nullspace described above. The typical way to eliminate this nullspace effect in A-EFIE is to use the deflation method or manually assign the charge neutrality condition by decreasing the number of the charge basis. Recent findings show that, by using the constraint preconditioner, the electrostatic nullspace effect is naturally suppressed since the small eigenvalues are shifted away from zero. Spectral analysis and physical reason of this electrostatic nullspace will be presented.