Acceleration of the Fully Numerical Evaluation of Galerkin Interactions on Surface Elements

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***Abstract*—In this paper we propose a scheme to treat, as a whole, the 4-D reaction integrals appearing in the Method of Moments. The approach is based on application of the surface divergence theorem to both source and test integrals, together with an appropriate integration reordering and variable transformations. To accelerate the convergence of numerical integration of all the integrals, contour integrals, variable transformations are applied to both the source and test contour integrals.**

Keywords— integral equations; moment methods; numerical analysis; singular integrals

I. Introduction

The accurate computation of singular and near-singular potential integrals, present in the Method of Moments system matrix, is of crucial importance in developing numerical codes able to accurately model and predict the behavior of problems in electromagnetics. For an accurate and efficient implementation of moment method solutions of surface integral equations, it is essential to numerically evaluate the double surface reaction integrals. Recently, in [1] the advantages of treating 4-D reaction integrals (source and test surface integrals) in their entirety were demonstrated. However, that approach was limited to self, edge-, or vertex-adjacent and well-shaped triangular elements owing to the use of translation of the triangular integration domain into equilateral triangle coordinates.

In the present paper, the radial integrals are instead performed directly in configuration space, thus eliminating the restriction to well-shaped and touching elements, as required by mapping to the angle-distorting equilateral coordinates.

The present paper is a natural extension of work presented previously. In [2]-[3] the approach has been developed for coplanar elements and for potential kernels with $1/R$ singularities. In [4] we proposed a scheme to treat, as a whole, the 4-D reaction integrals appearing in the Method of Moments with arbitrary oriented test/source triangles. However, in [4], acceleration techniques to further smooth the resulting integrands were not yet available. As shown, e.g. in [4], the use of acceleration techniques employing appropriate transformations can further smooth the resulting integrands and hence accelerate their convergence.

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II. Formulation

Consider the following integral over the planar source triangle S' with unit normal $\hat{\mathbf{n}}'$ and $\mathbf{r}$ fixed:

$$I_{S'} = \int_{S'} F(\mathbf{r},\mathbf{r}')\,dS'. \tag{1}$$

Typically, $F(\mathbf{r},\mathbf{r}')$ takes the form

$$F(\mathbf{r},\mathbf{r}') = t(\mathbf{r})g(\mathbf{r},\mathbf{r}')b(\mathbf{r}'), \tag{2}$$

and where $t(\mathbf{r})$ is either a scalar or a vector component of a testing function, $b(\mathbf{r}')$ is similarly defined for a basis function, and $g(\mathbf{r},\mathbf{r}')$ is either scalar, or a scalar component of a vector or dyadic Green's function. Applying the surface divergence theorem, the integral in (1) can be rewritten as

$$I_{S'} = \int_{S'} \nabla'\cdot\mathbf{H}\,dS' = \oint_{C'} \mathbf{H}(\mathbf{r},\mathbf{r}'_{C'})\cdot\hat{\mathbf{u}}'dC', \tag{3}$$

where $\mathbf{r}'_{C'}$ is a point on the boundary C' of S', $\hat{\mathbf{u}}'$ is the external normal to the contour in the triangle plane, and

$$\mathbf{H}(\mathbf{r},\mathbf{r}'_{C'}) = \frac{\hat{\mathbf{D}}'}{D'_{C'}}\int_0^{D'_{C'}} F(\mathbf{r},\mathbf{r}')\,D'dD', \tag{4}$$

with $\mathbf{r}' = \mathbf{r}^0 + D'\hat{\mathbf{D}}'$, $\hat{\mathbf{D}}' = (\mathbf{r}'_{C'} - \mathbf{r}^0)/D'_{C'}$, $0 \le D' \le D'_{C'}$, $D'_{C'} = |\mathbf{r}'_{C'} - \mathbf{r}^0|$ and $\mathbf{r}'_{C'}$ a point on C'. The point $\mathbf{r}^0$ is an arbitrarily located local polar coordinate origin in the plane of

S' used in applying the divergence theorem; a different origin may be associated with each value of $\mathbf{r}$, which we can express as a mapping, $\mathbf{r}^0 = \mathcal{T}^-(\mathbf{r})$, the negative sign in $\mathcal{T}^-$ reminding us that values $\mathbf{r}$ in the observation plane are mapped to the source plane, i.e., in the direction of decreasing tilt angle between the planes (see Fig. 1).

Considering now an additional testing integration over the planar observer domain S, we write the *double* surface integral as

$$I_{SS'} = \int_S \int_{S'} F(\mathbf{r},\mathbf{r}')\, dS' dS \;=$$

$$= \oint_{C'} \hat{\mathbf{u}}' \cdot \left[\int_S \left(\frac{\hat{\mathbf{D}}'}{D'_{C'}} \int_0^{D'_{C'}} F(\mathbf{r},\mathbf{r}')\, D' dD' \right) dS \right] dC', \qquad (5)$$

where the interchange of integration order is permitted by the independence of the source and observation coordinates and their associated domains. Applying the surface divergence theorem once more to (5), we obtain

$$I_{SS'} = \oint_C \left[\oint_{C'} \left(\frac{\hat{\mathbf{u}} \cdot \hat{\mathbf{D}}}{D_C} \int_0^{D_C} \left(\frac{\hat{\mathbf{u}}' \cdot \hat{\mathbf{D}}'}{D'_{C'}} \int_0^{D'_{C'}} F(\mathbf{r},\mathbf{r}')\, D' dD' \right) D dD \right) dC' \right] dC \qquad (6)$$

where C is the boundary of S, $\hat{\mathbf{u}}$ is the external normal to C in the observer plane, $\mathbf{r} = \mathbf{r}'^0 + D\hat{\mathbf{D}}$, where $\hat{D} = (\mathbf{r}_C - \mathbf{r}_{C'}^{'0}) / D_C$ and $0 \le D \le D_C$, with $D_C = \left| \mathbf{r}_C - \mathbf{r}_{C'}^{'0} \right|$ and $\mathbf{r}_C$ is a point on C. The mapping $\mathbf{r}'^0 = \mathcal{T}^+(\mathbf{r}'_{C'})$ transforms a point $\mathbf{r}'_{C'}$ on C' to a local polar coordinate origin $\mathbf{r}'^0$ in the plane of S.

A convenient choice of transform pairs are the *rotational* projections $\mathcal{R}^{\pm\beta}$ of source points onto the observer plane and vice versa. Thus we choose $\mathcal{T}^{\pm} = \mathcal{R}^{\pm\beta}$, $\mathcal{R}^{\beta}\mathcal{R}^{-\beta} = \mathcal{I}$, where operator $\mathcal{R}^{\beta}$ rotates points through the angle β, considering this projection (6), can now be written as

$$I_{SS'} = \oint_C \oint_{C'} \left(\frac{(\hat{\mathbf{u}} \cdot \hat{\mathbf{D}})(\hat{\mathbf{u}}' \cdot \hat{\mathbf{D}}')}{D_C} \int_0^{D_C} \left(\frac{1}{D'_{C'}} \int_0^{D'_{C'}} F(\mathbf{r},\mathbf{r}')\, D' dD' \right) D dD \right) dC' dC \qquad (7)$$

since $(\hat{\mathbf{u}}' \cdot \hat{\mathbf{D}}')$ is independent of D. Taking into account the conclusions about the coplanar elements case [2], variable transformations have been applied to accelerate the evaluation of edge integrals.

III. Numerical Results

To analyze the accuracy of the 4-D reaction integral reported in (6), we examine the scalar potential interaction term in the Method of Moment discretization of the Electric Field Integral Equation (EFIE). We consider a pair of non-touching source and test triangles in each other's near field, as shown in the inset of Fig. 2, and that demonstrate the generality of the method. All the integrals are evaluated numerically comparing the standard Gauss Legendre (GL) quadrature scheme to a reference result obtained using the Double Exponential (DE) scheme [5].

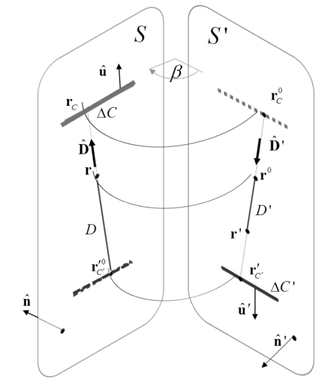

Fig. 1. The orientation of a pair of triangular elements in space and their respective projections.

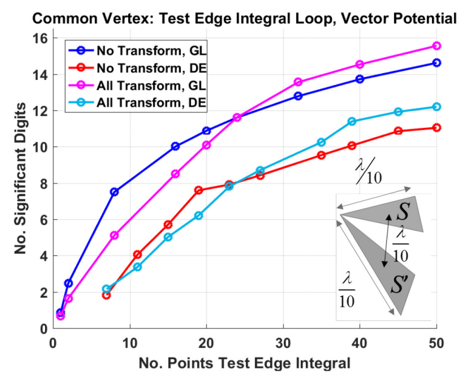

Fig. 2. Near-field convergence of contour integrals. Inset.- Orientation of a pair of triangular elements in space

IV. Conclusions

In the work outlined here we numerically validate the ability of the proposed approach to deal with the singularities of the type present in the MoM discretization of the Electric Field Integral Equation (EFIE) and related formulations.

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