Alternative Constraints for Divergence-Conforming Constrained Basis Functions

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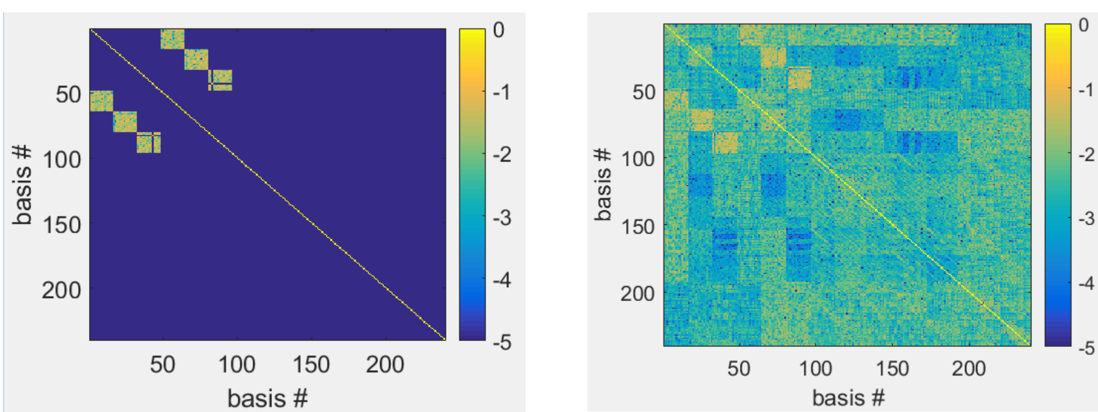
The constrained basis function method (N. Hendijani et al, IEEE Trans. Antennas & Propag., 63, 3111-3121, 2016) is a technique to generate arbitrary-order basis functions on mesh cells without specifying an analytic form *a priori.* Instead, bases are expanded in an underlying function set and forced to satisfy desired constraints on the cell(s) of support. The singular value decomposition is used to determine the necessary expansion coefficients so that the resulting bases satisfy the given constraints. Constrained bases on various cell types have been proposed and numerically characterized. In particular, the bases exhibit very good properties with respect to system condition number. However, the most basic set of constraints result in basis functions that do not explicitly separate (for divergence-conforming bases) into solenoidal and non-solenoidal spaces for higher-basis orders. In addition, with an appropriate choice of underlying function set, the bases will exhibit a high degree of orthogonality on reference cells, but the orthogonality rapidly deteriorates on skewed or curvilinear cells. For example, in Figure 1 is plotted the normalized log magnitude of the Gram matrix for basis order three, hexahedral constrained bases on (a) the unit reference hex and (b) a skewed hex, and the loss of orthogonality is easily observed.

In this paper, the constrained basis function method for producing divergence-conforming bases will be outlined for various cell types with appropriate underlying function sets and basic constraints discussed. Other sets of constraints will then be explored with a particular focus on formulating constraints that produce a quasi-Helmholtz decomposition of the higher-order bases into solenoidal and non-solenoidal sets. Furthermore, modifications of the constrained basis method to mitigate the loss of orthogonality on skewed or curvilinear cells will be investigated. Numerical characterization of the various basis sets will be performed with particular emphasis on error convergence, system conditioning, and mutual orthogonality of basis functions.



(a) (b)

Figure 1. Gram matrix normalized log magnitude for divergence-conforming hexahedral constrained bases on (a) the unit reference hex and (b) a skewed hex.