Metaplatforms for Solving Integral Equations with Waves

Nasim Mohammadi Estakhri*, Brian Edwards, and Nader Engheta
University of Pennsylvania, Department of Electrical and Systems Engineering
Philadelphia, PA 19104, USA, esmo@seas.upenn.edu

The concept of metamaterials —i.e. artificially designed materials with properties that are not readily available in nature— has opened up a wide range of possibilities and potential applications in many areas of wave engineering, and in particular in the electromagnetics and optics. Recently, our group proposed a metamaterial platform for performing analog mathematical operations such as differentiation, convolution and integration, operating on "light" signals (A. Silva *et al.* Science 343, 160-163, 2014). This technique allows compact and inherently fast solution of a *forward* problem. For instance, an integrator block can be designed to transform an arbitrary impinging wave profile $I_{in}(x)$ into $g(x)=\int K(x-x')I_{in}(x')dx'$.

In the present work, we propose a novel metamaterial-based platform for solving the *inverse* problems with waves, namely, to solve linear integral equations, even when their kernels are translationally variant. Let us consider a generic integral equation in form of Fredholm equation of second kind, $g(x)=I_{in}(x)+\int K(x,y)g(y)dy$, to be solved for an arbitrary input $I_{in}(x)$. Here, $K(x,y)$ is the kernel of equation and in general it can be translationally variant: $K(x,y)\neq K(x-y)$. In order to solve this equation by means of electromagnetic waves we propose a network consisting of the hybrid metastructures functioning as the kernel combined with the feedback transmission lines or waveguides to create a recursive path for the signal. As the wave propagates in such a "closed-loop" hybrid system and interacts with the impinging signal, the desired solution $g(x)$ is generated in the output of the hybrid metamaterial-waveguide system. Further details on the numerical and design criteria for equation-solving metamaterial networks will be discussed in our presentation. As an example, Fig. 1 illustrates our numerical solution of the above equation for $K(x,y)=0.7e^{-(x^2+y^2)}$ considering 20 sampling points across $x\in[-3,3]$.

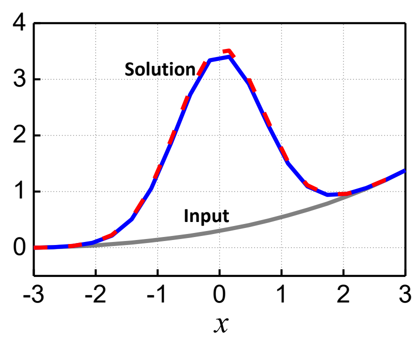

Figure 1. Solving above integral equation for $K(x,y)=0.7e^{-(x^2+y^2)}$ using our proposed metamaterial-waveguide feedback network: The arbitrarily chosen input, calculated solution $g(x)$, and the "expected exact" solution derived from direct matrix inversion are indicated by gray, blue and dashed red lines, respectively.