A Perfectly Matched Layer Integrated with Boundary Integral Equation to Serve as the Absorbing Boundary Condition

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Spectral element method (SEM) can model large scale problems accurately and efficiently with only a few elements. However, these problems are usually open-region in reality and therefore, good absorbing boundary conditions are required to truncate the computational domain properly. One candidate is the perfectly matched layer (PML) (J. Berenger, J. Comput. Phys., 114(2), 185-200, 1994), which can maintain the sparsity of the resultant matrices but add many physically redundant elements to the computational domain and increase the dimension of resultant matrices dramatically, especially for a long and thin physical domain. The other candidate is the boundary integral equation (BI), which can keep the dimension of resultant matrices invariant but destroy the sparsity of them.

Figure 1: The struture diagram for PML-BI combined method.

To integrate their advantages, we propose a novel hybrid absorbing boundary condition, the PML-BI boundary condition. To illustrate its principle, we use a two dimensional model as shown in Fig. 1. The yellow region is a physical domain and the blue region is PML domains; the red boundaries Γ_B employ the boundary integral equation and the black boundaries Γ_P can be either PEC or PMC boundaries. Suppose the unknowns are vertical magnetic fields H_z, then we can have the weak form for the SEM as follows

$$\begin{aligned}&\int_\Omega [-(\vec{z}\times\nabla_t w_m)\cdot\overline{\overline{\epsilon}}_r^{-1}(\vec{z}\times\nabla_t\widetilde{H}_z)+w_m k_0^2\mu_{rz}\widetilde{H}_z]d\Omega\\ =\;&\int_\Gamma w_m(j\omega\epsilon_0\widetilde{E}_t-\vec{t}\cdot\epsilon_r^{-1}J_t)d\Gamma+\int_\Omega w_m j\omega\epsilon_0 M_z d\Omega+\int_\Omega(\vec{z}\times\nabla_t w_m)\cdot\epsilon_r^{-1}J_t d\Omega.\end{aligned}$$

where t represents the transverse plane, z represents the vertical direction, w_m is the testing function, $\widetilde{H}_z = H_z, \overline{\overline{\epsilon}}_r = \epsilon_r\cdot diag(\frac{e_y}{e_x},\frac{e_x}{e_y}), \mu_{rz} = \mu_r e_x e_y$, e_x and e_y are the PML's scaling factors. Notice that the term $\int_\Gamma w_m(j\omega\epsilon_0\widetilde{E}_t-\vec{t}\cdot\epsilon_r^{-1}J_t)d\Gamma$ is zero at Γ_P but nonzero at Γ_B. Therefore, we need another equation to relate $\widetilde{E}_t$ with $\widetilde{H}_z$ at Γ_B and we use one order homogeneous BI here

$$\frac{\Omega_b}{4\pi}H_z+\int_{\Gamma_B,R}H_z(t')\vec{z}\cdot[\nabla g(k_bR)\times\vec{t}(t')]dt' = j\omega\epsilon_0\epsilon_{rb}\int_{\Gamma_B}E_t(t')g(k_bR)dt'$$

where $\vec{t}$ is the tangential direction at the boundary, R is the distance between the observation point and the equivalent source point, g is the 2D scalar Green's function and subscript b represents background material outside the computational domain.