Improving Discontinuous Galerkin Method Contrast Source Inversion Microwave Imaging using a Hybridizable Forward Solver Formulation

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A time-harmonic Discontinuous Galerkin Method (DGM) forward solver offers significant modeling flexibility in the context of microwave imaging algorithms such as the Contrast Source Inversion (CSI) method. Standard DGM formulations, which discretize Maxwell's curl equations, support simultaneous simulation of both electric and magnetic fields in the presence of both complex permittivity and permeability target profiles (J. S. Hesthaven and T. Warburton, *Nodal Discontinuous Galerkin Methods*, 2007). Our recent work (I. Jeffrey et al., URSI GASS, 2014) has led to the development of a parallel 3D DGM-CSI imaging algorithm for simultaneous permittivity and permeability imaging. This algorithm supports independently-selected, variable, high-order representations of the fields, contrasts, and contrast sources, that combined with the DGM's support of high-order inhomogeneous background representations, results in a versatile imaging algorithm suitable for a variety of applications including magnetic-nanoparticle-enhanced biomedical imaging (B. Gennaro et al., IEEE Trans. Biomed. Eng., 58.9, 2011).

While flexible, DGM-CSI suffers from the computational overhead of the curl-equation DGM forward solver, which necessarily simulates all components of both electric and magnetic fields in each computational element. In this work we focus on converting the curl-equation DGM forward solver to a Hybridizable Discontinuous Galerkin (HDG) formulation (L. Li, S. Lanteri, and R. Perrussel, J. Comput. Phys., 256, 563-581, 2014). HDG forward solvers improve DGM performance using a two-step solution procedure. First, the equations enforced by the curl-equation DGM are recast as a global system of equations that can be solved to determine the tangential magnetic fields on each trace between elements in the computational domain. Solving this global system enables the second step of the algorithm, where a local system is used to recover both the electric and magnetic field components in each element from knowledge of the tangential magnetic fields over the element boundary. The primary advantage of the HDG formulation over the standard curl-equation DGM formulation relates to the size of the global system: for a pth order solution on a computational domain consisting of N tetrahedral elements, the DGM global system has dimensions $(p+1)(p+2)(p+3)N$, while the HDG global system size is reduced to approximately $2(p+1)(p+2)N$.

The necessary steps to reformulate DGM-CSI as HDG-CSI for electric and magnetic field imaging of complex electric and magnetic targets will be discussed, including the necessary changes to gradient calculations. Attention will be given to the fact that, within the context of CSI, the local HDG systems need only be solved for elements within the imaging domain and/or those elements required to evaluate fields at data collection points. The performance benefits of HDG-CSI over DGM-CSI will be presented for a variety of imaging problems.