Optimal Scaling Factors of One-Element Perfectly Matched Layer in Spectral Element Method

Runren Zhang* and Qing Huo Liu*
Duke University, Durham, NC 27705

The absorbing boundary condition is critical to model the unbounded electromagnetic problem like radiation, scattering and propagation in an unbounded space. Compared to other absorbing boundary conditions including scattering boundary condition and boundary integral equation, perfectly matched layers (PML) have both inexpensive computation cost and stable absorbing capability. Therefore, it is valuable to find the optimal PML's scaling factors to obtain the most accurate result for different basis function orders.

To study the PML's scaling factors, we develop a set of codes to model two-dimensional transverse electric field (TE) problems by spectral element method (SEM). The basis functions order can range from 1 to 10. To make both the formulations and codes compact and succinct, we generate a generalized weak form of wave equation (F. L. Teixeira and W. C. Chew, Microwave Opt. Tech. Lett., 17(4), 231-236, 1998), which works well for both physical domain and PML domain as follows,

$$\begin{aligned}&\int_\Omega [-(\vec{z}\times\nabla_t w_m)\cdot \overline{\overline{\epsilon}}_r^{-1}(\vec{z}\times\nabla_t \widetilde{H}_z) + w_m k_0^2 \mu_{rz}\widetilde{H}_z]d\Omega \\ = &\int_\Gamma w_m(j\omega\epsilon_0\widetilde{E}_t - \vec{t}\cdot\epsilon_r^{-1}J_t)d\Gamma + \int_\Omega w_m j\omega\epsilon_0 M_z d\Omega + \int_\Omega (\vec{z}\times\nabla_t w_m)\cdot\epsilon_r^{-1}J_t d\Omega.\end{aligned}$$

where t represents the transverse plane, z represents the vertical direction, ∇_t represents the derivative in terms of x and y, w_m is the testing function, $\widetilde{H}_z = H_z$, $\overline{\overline{\epsilon}}_r = \epsilon_r \cdot diag(\frac{e_y}{e_x}, \frac{e_x}{e_y})$, $\mu_{rz} = \mu_r e_x e_y$. e_x and e_y here are the PML's scaling factors and are either $\alpha + j\beta$ or 1.

To evaluate the PML's performance, we model a case where an infinite long magnetic line source is placed vertical to the plane in free space. The simulation region is a 2 m $\times$ 1 m rectangle discretized by structured square meshes with side lengths 0.05 m. The line source is placed at (-0.75 m, -0.25 m) and the outermost layer is PML. This problem has an analytical solution, and therefore we can evaluate the PML's performance by $err = 20\log_{10}\frac{|H_{boundary}^{SEM} - H_{boundary}^{analytical}|}{|H_{boundary}^{analytical}|}$, where $H_{boundary}$ is the magnetic fields at the boundary elements.

By fixing $N = 4$, we find the lowest err inside an appropriate range for α and β at frequencies from 300 MHz to 1000 MHz and then obtain a fitting equation and a set of parameters for it. By changing the order of basis functions, we can obtain different sets of parameters for the fitting equation. At last we will explain the fitting equation from both mathematical and physical perspectives.