

# **A Nyström-based Explicit Time Marching Scheme for Solving the Time Domain Magnetic Field Integral Equation**

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Transient electromagnetic scattering from perfect electrically conducting (PEC) objects can be analyzed by solving time domain integral equations (TDIEs). Indeed, in the last two decades, marching-on-in-time (MOT)-based TDIE solvers have become appealing alternatives to differential-equation solvers such as finite-difference time-domain and time-domain finite-element methods. Depending on the types of the time and space basis functions and the time step size, MOT schemes can be either implicit or explicit. Even though explicit MOT schemes do not call for a matrix inversion at every time step, they are not as stable as their implicit counterparts and require smaller time step sizes to ensure stability. To alleviate the disadvantages of the explicit MOT schemes, a new novel quasi-explicit MOT scheme has been developed to solve the time domain magnetic field integral equation (TD-MFIE) (H. A. Ulku et al., IEEE Trans. Antennas Propag., 61, 4120-4131, 2013). This scheme converts the TD-MFIE discretized using Rao-Wilton-Glisson (RWG) basis functions into a system of first order ordinary differential equations. This matrix system is then integrated numerically in time to yield the coefficients of RWG basis functions expanding the current density. Unlike the traditional explicit MOT schemes, this solver's time step size can be as large as that of its implicit counterparts without sacrificing accuracy or stability. This quasi-explicit scheme solves a matrix system involving a Gram matrix at every time step. Unlike the implicit solvers, the sparsity level of this system does not depend on the time step size. Consequently, it can be solved very efficiently using an iterative method even at low frequencies (when the time step size is large).

In this work, a fully explicit MOT solver is developed to solve the TD-MFIE. Unlike the above quasi-explicit solver, this new scheme uses the Nyström method for discretizing the TD-MFIE in space. The Nyström method employs higher-order vector spatial basis functions defined at a set of discrete points on a given discretization element to expand the unknown current density. Since the basis functions reside on separate discretization points, the resulting Gram matrix, and its inverse are block diagonal with two-by-two blocks. This Gram matrix is inverted and stored before the time marching starts. This makes the time marching very efficient since the "matrix inversion" needed by the quasi-explicit solver is now replaced by a single blocked-diagonal matrix-vector multiplication. Additionally, the explicit solver developed here is higher-order accurate both in space and time.

Numerical results, which demonstrate the proposed solver's accuracy, efficiency, and applicability, will be presented.