Efficient Guide-star Enabled Reconstruction of Scattering Matrices Via Alternating Minimization and Proximal Gradient Methods

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Knowledge of the electromagnetic (EM) and optical transmission characteristics of random media is vitally important in applications ranging from medical imaging to nondestructive testing and geophysical exploration, to name but a few. The forward and backward transmission characteristics of linear and reciprocal media are fully characterized by their $\mathbf{S}_{12} = \mathbf{S}_{21}^T$ transmission matrices, which conventionally are determined using repeated measurements of modal amplitudes of transmitted (output) fields for different incident (input) excitations. Unfortunately, many applications do not allow for the direct measurement of transmitted wave amplitudes as the media's output aperture is not readily accessible. Under such conditions, only backscattered waves, generated by the media in conjunction with scatterers residing in the output aperture, can be measured.

In this work, we propose a novel approach to retrieve $\mathbf{S}_{21}$ for a given random medium with the assistance of "guide stars." Here, "guide star" refers to a strong scatterer with known scattering characteristics that resides in the output aperture of the random medium. Guide stars are often used in astronomy to calibrate telescopes for tracking heavenly bodies. In a nutshell, guide stars generate backscattered fields containing information that enables the reconstruction of $\mathbf{S}_{21}$ from backscatter measurements. The guide star assisted $\mathbf{S}_{21}$ retrieval problem can be expressed as a matrix equation $\mathbf{S}_{21}^T \cdot \mathbf{S}_g \cdot \mathbf{S}_{21} \cdot \mathbf{X} = \mathbf{Y}$, where $\mathbf{X}$ refers to the incident wave, $\mathbf{Y}$ refers to the backscattered wave, and $\mathbf{S}_g$ is the guide star's backscattering matrix. Classical techniques for solving this equation for the $n \times n$ $\mathbf{S}_{21}$ require $O(n^4)$ memory and $O(n^6)$ CPU time, and are impractical when $\mathbf{S}_{21}$ is large. Here, we propose two iterative schemes to solve the above equation, leveraging alternating minimization (least square) and proximal gradient (relaxed convex optimization) methods. Both permit the efficient solution of the above equation using $O(n)$ guide stars and $O(n^4)$ CPU resources.

The proposed approach for reconstructing $\mathbf{S}_{21}$ is applied to 3D random media comprised of multilayer periodic slabs containing arbitrarily positioned metallic particles, and small corner reflectors that serve as guide stars. Numerical simulations demonstrate the successful and highly accurate retrieval of an $n \times n$ $\mathbf{S}_{21}$ using $O(n)$ guide stars.