

# Efficient Estimation of Parameter Sensitivities in the FDTD Method Using Automatic Differentiation

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Estimating the sensitivity of a device, antenna, or circuit response to the material properties and the physical dimensions is an important part of the engineering design process. In particular, sensitivity analysis can be used to determine the stability of the design to the inherent variabilities that arise during manufacturing processes. Additionally, local joint-sensitivity information allows numerical optimization techniques to converge with a reduced number of iterations. The partial derivatives of the solution with respect to the input parameters of interest can be estimated via a central-difference approximation. However, this requires an additional two simulations per parameter and cannot be used to compute joint sensitivities directly. Furthermore, the central-difference approximation is highly prone to numerical errors, particularly as the parameter step size decreases toward zero. Using a complex-step derivative can eliminate numerical precision errors caused by central-differences (C.D. Sarris and H.-D. Lang, Proc. IEEE-IMS 2015). However, this approach is difficult to apply when multiple parameter sensitivities need to be computed.

In this abstract we propose the use of automatic differentiation (AD) to efficiently estimate multi-parameter sensitivities in the widely used finite-difference time-domain (FDTD) method. AD computes the partial derivatives of any complex expression (such as a system of FDTD update equations) by repeatedly applying the chain rule until it can be expressed using the derivatives of the arithmetic operators and elemental functions. AD can be efficiently implemented by operator overloading without requiring extensive modification of the underlying implementation and can estimate multi-parameter sensitivities from a single simulation run. To illustrate this, Fig. 1(a) shows a two-stub microstrip filter designed for operation at X-band. As a first step to evaluate the sensitivity of this design, Fig. 1(b) shows the derivative of the transient field recorded at port 2, with respect to  $\epsilon_r$  of the substrate, computed using central-differences and AD applied to a 3D FDTD code. Good agreement is found.

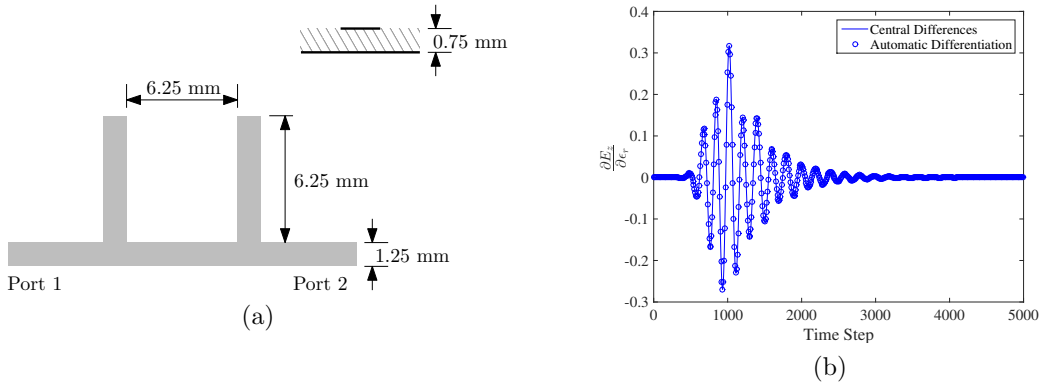


Figure 1: (a) X-band microstrip filter; (b)  $\frac{\partial E_z}{\partial \epsilon_r}$  computed with central differences and AD.