Green's Function Computation Using Lattice Sums for Leaky-Wave Analyses

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The full-wave numerical solution of electromagnetic problems with periodic dielectric or metallic structures requires the computation of periodic Green's functions. Specifically, if a cylindrical two-dimensional problem is considered, with period d and propagation constant k_{x0} along the x direction, a well-known periodic free-space Green's function can be formulated:

$$G(\Delta x, \Delta y) = \frac{1}{4j} \sum_{n=-\infty}^{\infty} H_0^{(2)}\left(k\sqrt{(\Delta x - nd)^2 + \Delta y^2}\right) e^{-jnk_{x0}d} \qquad (1)$$

where $H_0^{(2)}$ is the 0-th order Hankel function of the second kind, k is the free-space wavenumber, and $\Delta x, \Delta y$ are the differences between observation and source coordinates. Unfortunately, the extreme slow convergence of this series is not compatible with a practical numerical solution of a periodic problem. Moreover, if a complex mode (i.e., k_{x0} being a complex wavenumber, such as in leaky waves and in lossy structures) travels along the structure, the series (1) is not a valid representation for the Green's function, since its terms are exponentially growing as n goes to $+\infty$ or $-\infty$. A typical approach for the accurate and efficient evaluation of (1) is based on an appropriate formulation of the Ewald method (F. Capolino et al., IEEE Trans. Antennas Propag., 53, 2977–2984, 2005).

The computation of (1) can be made easier by recurring to a formulation in terms of Lattice Sums (LSs) (K. Yasumoto *et al.*, IEEE Trans. Antennas Propag., 47, 1050–1055, 1999). LSs are complex coefficients, namely, series involving higher-order Hankel functions, which uniquely characterize a periodic arrangement of objects and are independent of the polarization of the incident field, observation points, and the individual configuration of the scatterers, thus representing a significant advantage and a fundamental tool in scattering problems for discrete periodic structures. In particular, the LSs are widely used when scattered and guided waves in one-dimensional arrays of circular cylinders or crystal fibers are studied by means of the multipole and the T-matrix approaches (C. Y. Li *et al.*, IEEE Trans. Antennas Propag., 63, 3168–3178, 2015). Unfortunately, also the evaluation of LSs is difficult, due to the occurrence of slowly convergent series. Considerable efforts have been devoted to develop methods for the evaluation of LSs (C. M. Linton, SIAM Review, 52, 630-674, 2010). However, a purely real phase shift among adjacent cells has always been considered so far.

In this work, the computation of the LSs is extended for the first time to the case of general complex waves. Higher-order Ewald representations are suitably used for the accurate and efficient evaluation of each LS, which has been split into spectral and spatial Ewald series. The Gaussian asymptotic behavior of both higher-order Ewald series involved in the LSs grants a fast convergence even in the presence of complex wavenumbers, whereas the proper and improper nature of the field can be taken into account by suitably determining the transverse field behavior of each space harmonic in the relevant spectral Ewald series.