A HSS-Type Butterfly-Based Direct Integral Equation Solver for 3D Perfect Electrically Conducting Objects

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Butterfly-based direct integral equation solvers are rapidly gaining ground against FMM-based iterative solvers or low-rank-based direct solvers for large-scale electromagnetic problems that are inherently ill-conditioned or involve many right-hand sides. We recently developed low-complexity butterfly-based direct integral equation solvers for analyzing scattering from 2D and 3D, perfectly electrically conducting and homogenous dielectric objects (Guo et. al., in *URSI Digest*, 2015; Guo et. al., *IEEE Trans. Antennas Propag.*, 2017). These solvers construct a hierarchical block LU factorization of the impedance matrix by leveraging randomized schemes to construct butterfly-compressed sums and products of previously computed butterfly-compressed partial LU factors. The resulting solvers exhibit $O(N\log^2 N)$ memory and $O(N^{1.5}\log N)$ CPU complexities for large-scale scattering problems involving several million unknowns. That said, the computational efficiency of these solvers can be further improved. (i) The hierarchical LU factorization process does not compress blocks representing near-field interactions in the impedance matrix or their block LU factorization. (ii) The randomized butterfly scheme either exhibits unsatisfactory convergence behaviors or suboptimal CPU complexity.

In this work, we present a new butterfly-based direct solver that further improves the computational efficiency of the above-mentioned solvers for analyzing scattering from 3D perfect electrically conducting objects. The proposed 3D solver builds on a recently developed hierarchically semi-separable matrix (HSS)-type butterfly-based 2D solver (Liu et. al., *IEEE Antenn. Wireless Propag. Lett.*, 2017). The proposed solver factorizes the impedance matrix as a product of sparse factors, each consisting of off-diagonal blocks representing interactions between adjacent scatterers. The corresponding sub-blocks representing interactions between judiciously selected adjacent sub-scatterers are extracted to allow efficient butterfly compression of these blocks. Furthermore, the compression and inversion process leverages a newly developed optimal-complexity randomized butterfly scheme to reduce CPU complexity. The resulting HSS-type 3D direct solver requires $O(N\log^2 N)$ memory and quasi-linear CPU resources when applied to electrically large objects. The computational efficiency of the proposed solver will be compared with the above-referenced solvers that leverage LU factorization through numerical examples involving several million unknowns.