Fast back projection algorithm for circular SAR processing

Yi Liao, Wen-qin Wang, Huaizong Shao
The School of Communication and Information Engineering
University of Electronic Science and Technology of China
Chengdu, 611731, PRC
yliao@uestc.edu.cn

***Abstract*—This paper presents a fast back projection algorithm for circular synthetic aperture radar (CSAR) imaging. With the help of geometry analysis, the algorithm converts the circular aperture data into subimages. And a special polar coordinates system is adopted in the subimage fusion to make the whole process more efficient. Comparing with the direct back projection method, the proposed one improves the computational efficiency significantly. Experimental results demonstrate the accuracy and effectiveness of the algorithm.**

I. Introduction

Circular synthetic aperture radar (CSAR) is an important earth observation tool with its capability of multi-aspect observation, high resolution and 3D imaging [1]. Different from the conventional straight line SAR imaging, CSAR travels along a circular trajectory in a certain height plane. However, the circular track also brings challenges to the imaging. The classical frequency-domain based algorithms, e.g., range-doppler algorithm (RDA), chirp scaling algorithm (CSA) and polar format algorithm (PFA) cannot be directly employed into the CSAR processing. Soumekh gives a wavefront reconstruction method based on the Fourier decompositon of the Green's function [1], but it is hard to expand its application and exploit the motion compensation method. Back projection (BP) algorithm [2] seems to be a good approach for arbitrary geometry SAR imaging, whereas the big computational burden makes this approach too time consuming.

Fast factorized back projection (FFBP) algorithm [3] is a fast time domain approach that increases the computational efficiency by means of segmenting the data into subaperture SAR data and merging the subdata on polar grids to obtain the final image. Most FFBP algorithms are based on the linear trajectory [4]. In this letter, a fast BP algorithm (FBP) tailored for CSAR is proposed. By studying the circular flight configuration, the geometry of the circular subaperture is used in the image merging and interpolation kernels, making the proposed algorithm precise and efficient.

II. Signal model

The geometry of Circular apertures of CSAR is presented in Fig. 1. The radar follows the circular track with radius r_a. Assume that A is radar location where its azimuth angle is zero and the coordinates of B are (r_a, θ, H), $\theta \in [0, 2\pi)$. H is height of the platform. ω is the angular speed, t_a is the slow time and $\theta = \omega t_a$. P is an arbitrary target point in the scene whose coordinates are $(r_p, \theta_p, 0)$.

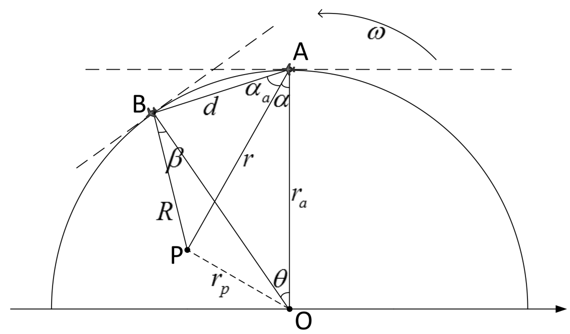


Fig. 1. The geometry of circular apertures.

Let the transmitted signal be linear frequency modulated signal, after demodulation and range compression, the collected data of target can be written as

$$s(t_r, t_a) = \sigma W_r(t_r - 2R(t_a)/c) w_a(t_a) \exp(-j4\pi f_c/c) \quad (1)$$

where t_r is the fast time, σ the backscattering coefficient of target, $R(t_a)$ the range between the radar and the target, f_c the carrier frequency. W_r and w_a are the range and azimuth envelopes, respectively.

A. Algorithm description

To perform the CSAR FBP algorithm, it is necessary to divide the aperture into subapertures first. Here the geometry of CSAR subaperture merging is provided to find a way of computing the new range and angle in the subimage merging. Additionally, the coordinates $(r, sin\alpha)$ are adopted in the processing instead of (r, α), increasing the algorithm efficiency.

In order to process the CSAR echo data, the first step is to partition the collected data and form coarse subimages. In Fig. 1, let point A be the aperture center point and B be the aperture center of next step. For A, the polar coordinates of target P are (r, α), and the polar coordinates change to (R, β) with new aperture center B. The relationship between the two coordinates need to be found. Based on the trigonometric identities, the range R can be expressed as

$$R = (d^2 + r^2 - 2r_1 d \cos \alpha_a)^{0.5} \tag{2}$$

with

$$d = (2r_a^2 - 2r_a^2 \cos \theta)^{0.5} \tag{3}$$

$$\alpha_a = \frac{\pi - \theta}{2} - \alpha \tag{4}$$

and the angle β can be presented as

$$\beta = \arccos \left(\frac{R^2 + r_a^2 - r_p^2}{2Rr_a} \right) \tag{5}$$

Back projection integral can be used here to obtain the subimages. And the back projection integral of echo data can be described in the wavenumber domain, i.e., the spatial frequency domain. The subimage magnitude $I_k(r, \sin \alpha)$ is coherently accumulated.

$$I_k(r, \sin \alpha) = \int_{\theta_k - \Delta\theta}^{\theta_k + \Delta\theta} s(r, \theta) \exp(-jK_c r) \mathrm{d}\theta \tag{6}$$

where θ_k denotes the azimuth angle with respect to the kth subaperture, $\Delta\theta$ means the coverage of the subaperture azimuth angle, $K_c = 4\pi/\lambda$ is the wavenumber and λ is the wave length of the carrier frequency. Different from the conventional polar grid system (r, α), in the proposed algorithm, the $(r, \sin \alpha)$ coordinate system is used to reduce the redundant computation of trigonometric functions in the processing.

After achieving the coarse subimages, the next step is subaperture fusion. In the new subaperture local polar coordinate system, every pixel's original location can be calculated by taking advantage of equation (2). Here the Nyquist sampling requirements should be satisfied to avoid ambiguity. The final image will be obtained with all the subspectrum merged together.

B. *Computational cost and analysis*

The computational load of proposed algorithm is analyzed in this section. Fig. 2 depicts the computation load of the two algorithms with the total image pixels $N = 16384$. The number of subapertures k varies from 1 to 1024. It can be found clearly from the figure that, as the subapertures increases, the total computational load will decrease rapidly. Comparing with the traditional BP algorithm, the computation complexity of FBP becomes lower and the efficiency improves significantly.

III. Simulation Results

Fig. 3 shows the simulated imaging results. It can be seen from Fig. 3(a) that all five points are well focused. Fig. 3(b) depicts the one-dimensional IRF of one of the target point located at $(0, 20)$. From the figure, we can find that the proposed FBP algorithm can deal with the CSAR imaging precisely and efficiently.

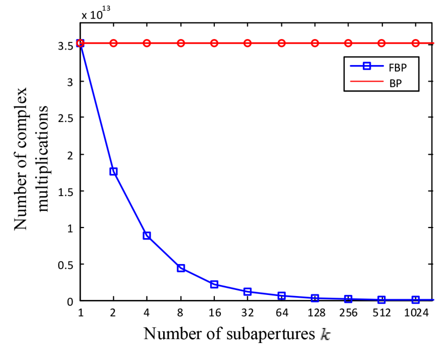
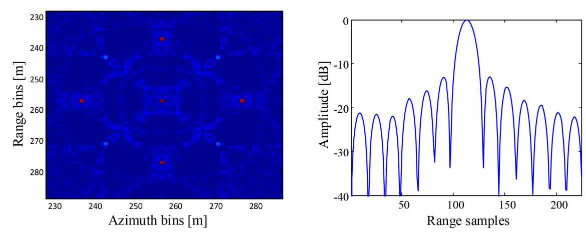


Fig. 2. Number of complex multiplications as subaperture number k increases.



(a) (b)

Fig. 3. Imaging performance. (a) Five points imaging results for proposed approach. (b) One dimensional IRF of the point target.

IV. Conclusion

We have analyzed the CSAR geometry and designed a fast back projection algorithm based on the CSAR configuration. A sin domain coordinates system is introduced to optimize the computation. The simulated experiment validates the focusing accuracy of the proposed approach and the computational superiority over the conventional direct BP algorithm.

ACKNOWLEDGEMENT

This work was supported in part by The National Natural Science Foundation of China under Grant 61571081 and in part by the Sichuan Technology Research and Development Fund under Grant 2015GZ0211.

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