A Study of Coherent Gain Degradation due to Node Vibrations in Open Loop Coherent Distributed Arrays

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Abstract—**One of the primary challenges in implementing open-loop coherent distributed arrays is the requirement to measure the separation between platforms to a fraction of a wavelength. For aerial platforms operating at microwave frequencies, it is thus important to understand the effects of platform vibration, in particular how often the range measurement must be updated in order to ensure that vibration does not sufficiently move the platforms such that the coherent gain is degraded. This paper analyzes worst-case vibration profiles for various aircraft types, showing that update rates of only 200 Hz are sufficient to ensure negligible reduction in coherent gain from platform vibration.**

I. Introduction

Coherent distributed arrays (CDAs) consist of numerous single platform wireless systems (called nodes) that are appropriately coordinated such that their wireless operations are coherent at the wavelength level. Such arrays can achieve significant gains in transmit power and spatial diversity compared to single-platform systems. While other efforts have focused on the development of closed-loop arrays, where feedback from the targeted location is provided to aid in coordinating the nodes in the array [1,2], this work focuses on arrays operating in an open-loop format. Open-loop coherent distributed arrays, where the array self-aligns without external inputs from the intended target location, are particularly amenable to a wide range of wireless applications, including remote sensing and radar [3]. Open loop CDAs behave as a distributed phased array and can thus steer the beam at any direction without the aid of an external signal. Such an open loop CDA system may find application in the future on small satellite platforms or on a group of unmanned aerial vehicles designed to study the moisture content of the soil which would prove beneficial for agricultural purposes. Designing an open loop CDA is challenging due to the requirements on inter-node coordination. Stemming from the need to steer a beam with phase or time delays, challenges are in the form of achieving an appropriate phase offset and wirelessly locking the frequency at each node. The measured internode range accuracy should be below $\lambda/15$ in order to achieve more than 90% coherent gain over the ideal [3] where λ is the wavelength of the transmitted signal by the array, and not the signal used for the inter-node coordination, which may be any frequency. In [4], a novel method of inter-node ranging using spectrally sparse microwave signals achieving ranging accuracies below 1 mm was developed and used in the first open-loop CDA demonstration.

In this paper we study the degradation of coherent signal at a far field due to platform vibrations which contributes to error in internode ranging. Analysis of this error term is of prime importance in order to maintain the coordination between platforms. In the next section we present a mathematical model of the signal received from a coherent distributed array taking into account the error in internode measurements caused by platform vibrations.

II. Signal Model

A simple uniform linear array with N elements placed at a distance d from each other has been considered for the purpose of studying the effects of platform vibration on coherent signal at far field. With perfect alignment and position measurements the ideal received signal in the far field is

$$s_i = C\sum_{n=1}^{N} e^{-j(n-1)\psi}, \tag{1}$$

where C is a constant and includes the amplitude of the signal. The time variation $e^{-j\omega t}$ term has been suppressed. In presence of internode ranging error δd due to platform vibration, the relative phase term is given as

$$\psi = k(d+\delta d)\cos\theta + \beta, \tag{2}$$

where $0° \leq \theta \leq 180°$ represents the azimuth angle and β is the progressive phase lead current excitation relative to the preceding one. In order to have maximum radiation of an array directed to the steering angle θ_0, the phase excitation β between the elements is calculated as

$$\beta = kd\cos\theta_0 \tag{3}$$

Substituting (2) in (1) gives the required received signal s_r. To study the effect of the error term, relative gain G_c is evaluated and is given by

$$G_c = \frac{|s_r s_r^*|}{|s_i s_i^*|}. \tag{4}$$

It can be seen that when the error due to platform vibrations is sufficiently small, G_c is approximately unity.

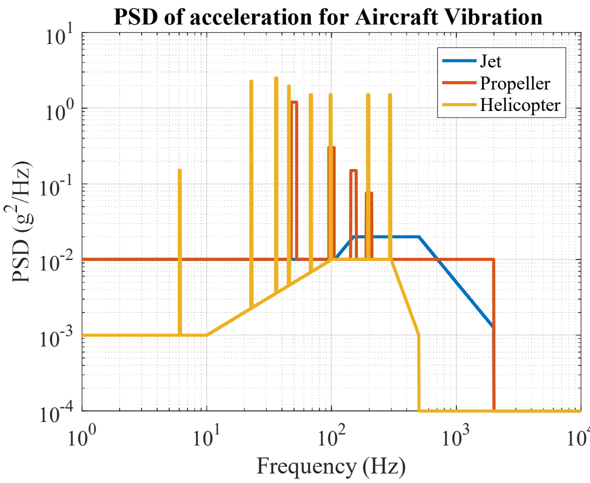

Fig. 1. Power spectral density of acceleration for a jet, propeller aircraft and helicopter [5].

To assess anticipated effects of vibrations on aerial platforms, we use maximum expected power spectral densities (PSD) of acceleration for different types of aircraft as defined in [5]. The PSD of acceleration in Fig. 1 describes the typical acceleration levels that a jet aircraft cargo, a propeller aircraft, and a helicopter are exposed to. For random vibration analysis, the PSD of acceleration is expressed in g^2/Hz. The amplitude spectral density (ASD) can be obtained by taking the square root of the PSD. The root mean square (RMS) displacement expressed in meters can then be calculated from the ASD as outlined in [5]. The RMS displacement represents the statistical uncertainty in the position of a body caused due to vibration. Since our analysis is based on measuring the degradation of signal with respect to the maximum possible error that may be induced in the phase term, heavier objects such as aircrafts have been considered to model vibrations of an airborne body. Each element of the array operates at a frequency f_c. Additionally, elements measure their inter-node range pair-wise at a specified update rate. The RMS displacement drops to zero the moment information from one element reaches the other, since perfect range knowledge is achieved at that time. Thus a higher update rate would mean a decreasing RMS displacement.

III. ANALYSIS

A 10 element array with a spacing of 100λ between the elements was analyzed, with the steering angle (S.A.) of 0° to account for the maximum possible phase difference. An increase in the update rate should diminish the uncertainty in the internode range as was discussed previously. Reduced displacement implies near-unity relative gain for higher update rates, as can be seen in Fig. 2, which confirms our initial prediction. The following observations can be made: (1) The random oscillations at lower update rate can be attributed to the error in the phase term caused due to platform vibrations. But at a higher update rate the gain settles down to 0 dB and thereafter remains flat in all three cases. (2) At a higher operating frequency f_c=100 GHz, as expected, the oscillations are more random, extends way beyond the point where f_c=1 GHz attains near unity gain and eventually settles down at a higher update rate. This is because at higher operating frequency the wavelength is much smaller than the maximum

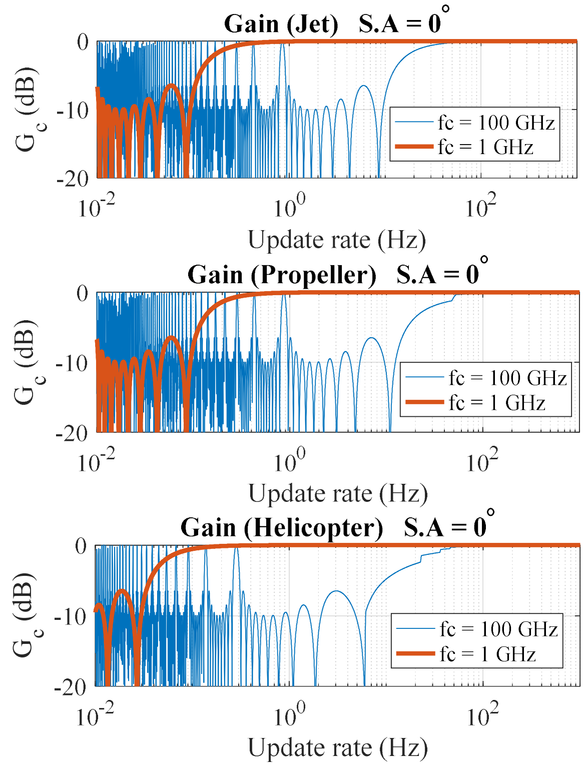

Fig. 2. Relative gain G_c at 1 GHz and 100 GHz versus update rate at a steering angle (S.A.) of 0°.

RMS displacement. (3) For an update rate more than 200 Hz, a 0 dB gain can be expected for any applications in the millimeter and microwave spectrum in worst-case scenarios without having to consider errors caused due to platform vibration as is evident from all three figures.

IV. CONCLUSION

The internode coordination requirements, in case of an open loop coherent distributed array, are challenging to achieve. This paper analyzed the problem of signal degradation caused due to platform vibration by considering a simple linear array, showing that updating the inter-node range measurement at a rate of 200 Hz or greater will eliminate any signal decoherence due to platform vibration. Future work will include extending the linear array to a randomly distributed 2D array along with considerations for error due to platform orientation, as well as other nontrivial factors such as time alignment and Doppler shift in case of fast moving platforms.

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