Third Order Modal Degeneracy in Waveguids: Features and Application in Amplifiers

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***Abstract*— We investigate a particular degeneracy condition in the dispersion of coupled periodic structures causing three eigenmodes to coalesce under proper coupling between eigenmodes supported by the system. This condition is referred to as a stationary inflection point (SIP) which is a third order degeneracy. We also show that by tuning the coupling parameters in the coupled periodic structure, the dispersion can be engineered to exhibit a slight positive or negative group velocity around the SIP frequency rather than zero group velocity. In particular, such regime provides a three-mode synchronous operation with an electron beam which is potentially beneficial for enhancing the efficiency of the traveling wave tubes (TWTs). Therefore, we use the proposed regime to design a SIP-based TWT where we show a gigantic gain enhancement compared to conventional single-mode Pierce-like TWTs.**

I. INTRODUCTION

Periodic structures have been utilized in many RF devices due to their unique dispersion properties [1], [2]. In particular, degeneracy may exist in a periodic structure where the eigenstates of the system coalesce and form a single degenerate periodic eigenstate. This phenomenon can be classified in general as an exceptional point of degeneracy (EPD) in the dispersion diagram. Here, we investigate the special case of third order degeneracy known as stationary inflection point (SIP) [3] in the dispersion diagram of a multimode waveguide where three electromagnetic modes coalesce to a single mode. Fig. 1(a) shows the schematic of a three coupled transmission lines (CTLs) which can feature SIP in its dispersion relation. The typical dispersion diagram of the three CTLs is also shown in Fig. 1(b). For the sake of simplicity, we have here omitted the branches with complex wavenumber so that only modes with real wavenumbers are shown (blue lines). Around the SIP the dispersion diagram is approximated by $(\omega-\omega_{\mathrm{SIP}})\propto(k-k_{\mathrm{SIP}})^3$ where ω_{SIP} is the angular frequency at which three modes coalesce and k_{SIP} is the Bloch wavenumber at the SIP. Note that $k_{\mathrm{SIP}}\neq\pi/d$ which means that the SIP does not occur exactly at the band edge of the Brillouin zone. This phenomenon is shown to bring some intriguing features that is desirable for high power electron beam driven TWT amplifiers.

II. SLOW-WAVE SRUCTURES WITH SIP

In this section, we exploit three CTLs that mimic the behavior of a waveguide with SIP as shown in Fig. 1(a). The details as well as the parameters of the periodic three CTLs are provided in the Appendix and its dispersion diagram (the real branches of $\omega-k$ diagram) is plotted in Fig. 1(b).

One important feature of the SIP is that the slope of the dispersion diagram can be engineered by tuning the CTLs distributed coupling parameters such that we have a zero, small positive, or a small negative group velocity ($\mathrm{v}_g=\partial\omega/\partial k$) around the SIP frequency. This may lead to the development of wideband amplifiers using the positive slope case or to the development of the low-threshold oscillators using the negative slope one. The dispersion diagrams for the three different cases mentioned above near the SIP frequency are plotted in Fig. 2(a). The CTL parameters for each case are also provided in the Appendix. We investigate an important application of the proposed concept in novel amplifier design based on a slow wave structure (SWS) with SIP interacting with an electron beam.

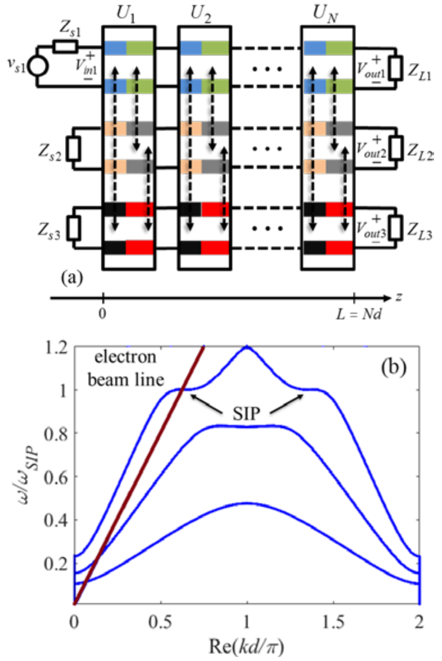

Fig. 1 (a) Schematic of a periodic CTL with the length of $L=Nd$ consisting of three lines, and two segments in each unit cell. (b) Dispersion relation of an infinitely long periodic 3CTL having SIP at angular frequency ω_{SIP}, including the electron beam line that illustrates the proposed three-mode synchronization regime for TWT applications.

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The potential gain enhancement in TWTs, operating near the SIP [3] in the cold structure, relies on a novel operational principle referred to as *three-mode interaction* which promises unique features not present in conventional single mode TWTs. We use the extension of the 1D linearized Pierce theory to CTLs coupled to a single electron beam as introduced in [4]–[6]. The three mode synchronous operation requires that the three modes at the SIP would constructively engage in the interaction with the electron beam only if they all have the phase velocity $\omega_{\mathrm{SIP}} / k_{\mathrm{SIP}}$ at ω_{SIP} that is coincident with the average beam velocity u_0. This is demonstrated graphically in Fig. 1(b) by the intersection of the "cold" SWS dispersion and the beam line dispersion $\omega = u_0 k$ at $\omega = \omega_{\mathrm{SIP}}$. In order to investigate amplification and gain of the proposed three CTLs-based TWT, we consider a finite-length SWS composed of *N* unit cells whose total power gain transfer function is defined as $P_{G,tot} = \sum_i P_{L,i} / P_{s,\max}$, where $P_{s\,\max}$ is the maximum available power at the input terminals of the first TL and $P_{L,i}$ is the power delivered to the output terminals of the *i*-th TL. We plot the power gain of the proposed tilted SIP amplifier (having a small positive group velocity) (in dB) versus the length of the structure (number of unit cells *N*) and compare to that of the single TL having the same impedance and length when the beam current is 150 mA. The result of the Fig. 2(b) shows a tremendous advantage for the proposed amplifier in terms of power gain as *N* increases.

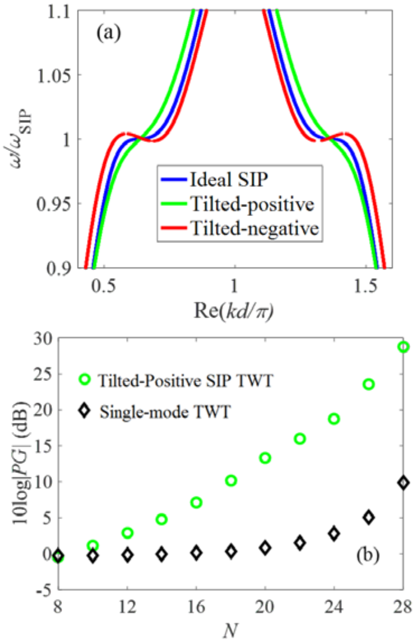

Fig.2. (a) Engineering the slope of the dispersion diagram to have a zero, a positive, or a negative group velocity (slope) around the SIP. (b) Power gain in dB versus the number of unit cells, *N*, for the tilted-positive scheme and 1TL model having beam current of 150 mA.

III. CONCLUSION

We have reported the existence of SIPs in coupled waveguides modeled using three CTLs. The prospective application of this concept is utilized for wideband power amplifiers while maintaining high gains. We have also shown that the SIP can be tuned to have positive or negative group velocities which results in performance improvements depending on the application that we will investigate further in the future. We have also shown the advantage of the proposed scheme over a single-mode Pierce-like TWT having same length and impedance characteristics that would lead to enhancing the efficiency of high power amplifiers.

APPENDIX: PARAMETERS USED IN SIMULATIONS

We consider a periodic 3CTL composed of unit cells made of two continuous uniform 3CTL segments. The CLT parameters are selected such that the average characteristic impedance is around 50Ω and the 3CTL develops an SIP around 1 GHz. Segment *A* of the CTL has the length of 10 mm and the following per unit length parameters: $L_{11} = 0.56\,\mu\mathrm{H/m}$, $L_{22} = 0.07\,\mu\mathrm{H/m}$, $L_{33} = 0.74\,\mu\mathrm{H/m}$, $C_{11} = 80.6\,\mathrm{pF/m}$, $C_{22} = 58\,\mathrm{pF/m}$ and $C_{33} = 151.6\,\mathrm{pF/m}$, and for the coupling parameters we have $C_{12} = 22\,\mathrm{pF/m}$, and $C_{23} = 0.7\,\mathrm{pF/m}$. Segment *B* has the length of 15 mm the following per unit length parameters: $L_{11} = 0.99\,\mu\mathrm{H/m}$, $L_{22} = 0.68\,\mu\mathrm{H/m}$, $L_{33} = 1.24\,\mu\mathrm{H/m}$, $L_{12} = 47$ nH/m, $L_{13} = 63$ nH/m, $L_{23} = 0.3\,\mu\mathrm{H/m}$, $C_{11} = 387\,\mathrm{pF/m}$, $C_{22} = 258\,\mathrm{pF/m}$, $C_{33} = 339\,\mathrm{pF/m}$, and $C_{12} = 25\,\mathrm{pF/m}$, $C_{23} = 111\,\mathrm{pF/m}$ $C_{13} = 145\,\mathrm{pF/m}$. For tilted-positive scheme the different parameters compared to ideal SIP in segment *B* have the values of $L_{12} = 78\,\mathrm{nH/m}$, $L_{13} = 2\,\mathrm{nH/m}$, and for tilted-negative the values of $L_{12} = 240\,\mathrm{nH/m}$, $L_{23} = 310\,\mathrm{nH/m}$, and $C_{13} = 97\,\mathrm{pF/m}$. The cut-off dispersion is provided by a series per unit length capacitance in each segment, with values of $C_{c1} = 0.6\,\mathrm{pF/m}$, $C_{c2} = 0.5\,\mathrm{pF/m}$, and $C_{c3} = 1\,\mathrm{pF/m}$. Losses are accounted in the TLs as series per unit resistances of 5.5 Ω/m in all TLs. These values are obtained based on cylindrical waveguide geometry supporting three coupled modes. The uniform 1TL used for comparison in Fig. 1 has the same total physical length as the 3CTL used for SIP case and per unit length capacitance and inductance of $C = 220\,\mathrm{pF/m}$ and $L = 0.55\,\mu\mathrm{H/m}$, and cut-off capacitance of $C_c = 0.3\,\mathrm{pF/m}$.

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