Comparison of Methods for Layered Spheroid Scattering Analysis: Application to BioEM Problems

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***Abstract*—Using spheroidal wave functions, scattering by layered spheroid can be solved by using separation of variables method (SVM) or extended boundary conditions method (EBCM). These two methods are analytically identical, while numerical implementation can result in differences in terms of convergence property as well as numerical stability. This work performs a comprehensive comparative assessment of the two approaches for BioEM applications.**

Keywords—extended boundary conditions method, separable of variables method, spheroidal wave functions, computational bioelectromagnetics

I. Introduction

Analysis of scattering from layered spheroids is useful for understanding and quantifying the field response of a human body [1] as well as separate tissues [2]. Spheroids of various aspect ratios can serve as useful low-fidelity models for various human tissues in various application. Although numerous general purpose numerical methods can be used for this purpose, the results obtained by analytical methods—whenever applicable—are, in general, significantly more accurate and cheaper to compute. Moreover, these results can be used as independent references for computational methods [3]; indeed, analytically evaluated references are essential for validating and benchmarking computational methods. However, the geometrical and electromagnetic characteristics of the adequate spheroidal models pose challenges to the existing analytical methods.

Analytical methods for analyzing spheroidal scattering have been investigated for many years [4][5]. Typically, spherical or spheroidal wave functions are used for the analysis of relatively small particles. Two popular such methods are the extended boundary condition method (EBCM) and the separation of variable method (SVM) [6], which can be viewed as the integral and differential forms, respectively [7], of the same interface condition problem. The EBCM solves the surface integral equations on interfaces between successive layers to construct transition matrices (T-matrices). In [1], it was implemented with spheroidal wave functions that conform to the surface, in order to avoid T-matrix instabilities for due to a large object aspect ratio [8][9]. However, the method experiences ill-conditioning for large problems, where many expansion terms are needed. In the more straightforward SVM, using spheroidal wave functions, the interface conditions are converted to a system of linear equations, which can be solved either recursively, i.e., layer by layer [6], or directly for all the layers at once [5]. While the two methods are analytically identical, the truncation and numerical implementation of the two methods lead to differences computational costs and convergence properties, for different problems. This calls for a careful comparison of the performance and applicability ranges of these two methods. Such a comparison in [10] referred only to spherical harmonics and the discussion was limited to scattering by small particles.

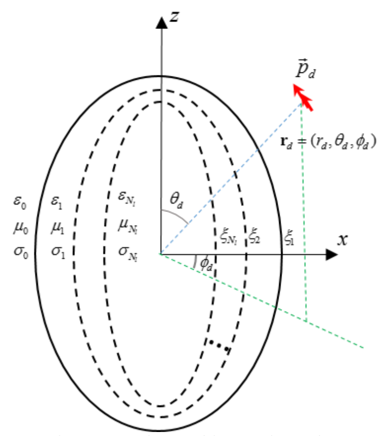

Figure 1: A layered isotropic prolate spheroid excited by magnetic dipole

This work extends the discussion in [10] and comparison between SVM and EBCM, using spheroidal wave functions, to BioEM applications, where large layered spheroids of high aspect ratios are of interest. A comprehensive comparison, in terms of the convergence rate, numerical stability, as well as computational cost is performed [1].

II. Presented work

Consider the scattering from an N_l -layered isotropic prolate spheroid. The fields are first decomposed by using Debye and Hertz potentials which could be expanded using spheroidal wave functions. In the SVM, as in the Mie theory for sphere scattering, a system of linear equations can be written by enforcing the continuity of the tangential fields and truncating the wave expansions with M^{T} for order m and $N^{\mathrm{T}}_{l,mn}$ for degree n, such that:

$$\mathbf{L}_{i-1,i}^{m} \cdot \begin{bmatrix} \mathbf{C}_{i-1}^{m} \\ \mathbf{C}_{i}^{m} \end{bmatrix} = \mathbf{0} \tag{1}$$

where $m = 0,...,M^{\mathrm{T}} - 1$, and $\mathbf{C}_{i}^{m}$ is a vector that stores the unknown coefficients with the number of $4N_{l,mn}^{\mathrm{T}}$ for each order m corresponding to the i^{th} layer [1].

The EBCM is based on the Huygens principle and is formulated using the surface integral equations on each interface, which provides a framework for recursively finding the coefficients for each layer. Similarly to SVM, the system of linear equations at the i^{th} interface can be written in matrix form as

$$\mathbf{T}_{i-1,i}^{m} \cdot \left[\mathbf{C}_{i}^{m} \right] = \mathbf{C}_{i-1}^{m} \tag{2}$$

where $\mathbf{T}_{i-1,i}^{m}$ is the transition matrix between $i-1^{\mathrm{th}}$ and i^{th} layer.

The solution of (1) and (2) for each of the layers in the recursive schemes is obtained by directly solving the system of linear equations, at a cost that is determined by the truncation number $N_{l,mn}^{\mathrm{T}}$. The number $N_{l,mn}^{\mathrm{T}}$ required for accurately solving a given problem is determined by its size and material properties: the larger the electric size, the greater $N_{l,mn}^{\mathrm{T}}$. However, the increase in $N_{l,mn}^{\mathrm{T}}$ leads to ill-conditioning of both $\mathbf{L}_{i-1,i}^{m}$ and $\mathbf{T}_{i-1,i}^{m}$ (not necessarily at the same rate) resulting in inaccuracies of the coefficients and non-convergence of the spheroidal function series.

At the conference, we will compare the convergence of the two proposed algorithms, for various aspect ratios, spheroid sizes, number of layers, and material properties, to determine their usefulness for modeling various types of tissues.

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