Multipath Signals in a Refractive Environment

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***Abstract*—Atmospheric ducting can cause electromagnetic (EM) waves to bend over earth's surface. While ducting can create temporary channels between long ranged receivers, these signals may also experience undesirable multipath fading. A sparse MIMO model is created for detection of such multipaths described by the arrival and departure angles of each path. Simulations suggest that it is effective at identifying the paths of signals with good signal to noise ratio (SNR), assuming few paths. The model is applied to southern California data to observe the multipath properties of ducted signals.**

I. INTRODUCTION

This paper deals with the detection of multipath signals traveling over the horizon through an evaporation duct. We develop a sparse MIMO signal model which can detect the paths taken through a duct as differentiated by the angle of arrival (AoA) and angle of departure (AoD) of each path. We then apply this model to MIMO data recorded in Southern California in a duct hotspot.

II. MULTIPATH SIGNAL MODEL

Consider N_T transmitters and N_R receivers, both positioned in a uniform linear array (ULA) with element spacing r and s, sending single frequency signals. Each of the transmitted signals is L samples. Each of P paths have a unique angle of departure (AoD) ϕ_p and angle of arrival (AoA) θ_p, and corresponding complex channel gain h_p, $p \in [1, \ldots, P]$.

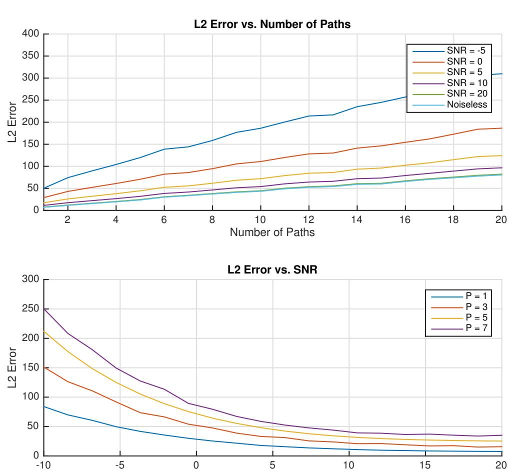

Fig. 1. ℓ_2 error of simulated signals of varying SNR and number of paths.

The received signal $\mathbf{y}(l)$ at time l is the sum of N_T transmitted signals multiplied by the respective AoA and AoD's of the P paths.

$$\mathbf{y}(l) = \sum_{p=1}^{P} h_p \mathbf{a}_R(\theta_p)\mathbf{a}_T(\phi_p)^H \mathbf{x}(l) + \mathbf{w} \tag{1}$$

where $\mathbf{x}(l) \in \mathbb{C}^{N_T}$ is the transmit signal vector in the frequency domain at time l, $\mathbf{w}$ is a vector of i.i.d Gaussian noise, and $\mathbf{a}_T(\phi)$ and $\mathbf{a}_R(\theta)$ are the transmit and receive steering vectors respectively defined $\mathbf{a}_T(\phi) = \exp[-j2\pi i \sin(\phi)r/\lambda]$, $\mathbf{a}_R(\theta) = \exp[-j2\pi k \sin(\theta)s/\lambda]$ [1] where $i \in [0, \ldots, N_T - 1]$, $k \in [0, \ldots, N_R - 1]$, and λ is the wavelength.

The possible AoD and AoA's are quantized into Q_R and Q_T discrete angles between $\pm\theta_{max}$ and $\pm\phi_{max}$ where $\theta_{max} = \sin^{-1}(\frac{\lambda}{2r})$ and $\phi_{max} = \sin^{-1}(\frac{\lambda}{2s})$ and λ is the wavelength at the carrier frequency. The quantized signal model is

$$\mathbf{y}(l) = \sum_{i=1}^{Q_R}\sum_{j=1}^{Q_T} h(i,j)\mathbf{a}_R(\theta_i)\mathbf{a}_T^H(\phi_j)\mathbf{x}(l) + \mathbf{w} \tag{2}$$

which is written in matrix notation

$$\bar{\mathbf{y}} = \mathbf{A}\bar{\mathbf{h}} + \bar{\mathbf{w}} \tag{3}$$

where $\bar{\mathbf{y}} \in \mathbb{C}^{N_R L}$ is the received signal $[\mathbf{y}^T(1), ..., \mathbf{y}^T(L)]^T$, $\bar{\mathbf{h}} \in \mathbb{C}^{Q_T Q_R}$ represents the channel gains, $\bar{\mathbf{w}} \in \mathbb{C}^{N_R L}$ is a Gaussian noise vector, and $\mathbf{A} \in \mathbb{C}^{N_R L \times Q_R Q_T}$ is the dictionary of paths

$$\mathbf{A} = \begin{bmatrix} \bar{\mathbf{a}}_{1,1}, ..., \bar{\mathbf{a}}_{Q_T,1}, & \ldots & , \bar{\mathbf{a}}_{1,Q_R}, ..., \bar{\mathbf{a}}_{Q_T,Q_R} \end{bmatrix} \tag{4}$$

$$\bar{\mathbf{a}}_{q_t,q_r} = \begin{bmatrix} \mathbf{a}_R(\theta_{q_r})\mathbf{a}_T^H(\phi_{q_t})\mathbf{x}(1) \\ \vdots \\ \mathbf{a}_R(\theta_{q_r})\mathbf{a}_T^H(\phi_{q_t})\mathbf{x}(L) \end{bmatrix} \tag{5}$$

$\bar{\mathbf{a}}_{q_t,q_r} \in \mathbb{C}^{N_R L}$ is a dictionary entry containing the returns of L samples with AoA and AoD equal to θ_{q_r} and ϕ_{q_t}.

We assume that the channel gains are constant over the L received samples, and $P \ll Q_T Q_R$. This is equivalent to assuming $\bar{\mathbf{h}}$ is P sparse, and should be recoverable through the sparse optimization

$$\hat{\mathbf{h}} = \min_{\bar{\mathbf{h}}} \frac{1}{2}||\bar{\mathbf{y}} - \mathbf{A}\bar{\mathbf{h}}||_2^2 + \mu||\bar{\mathbf{h}}||_1 \tag{6}$$

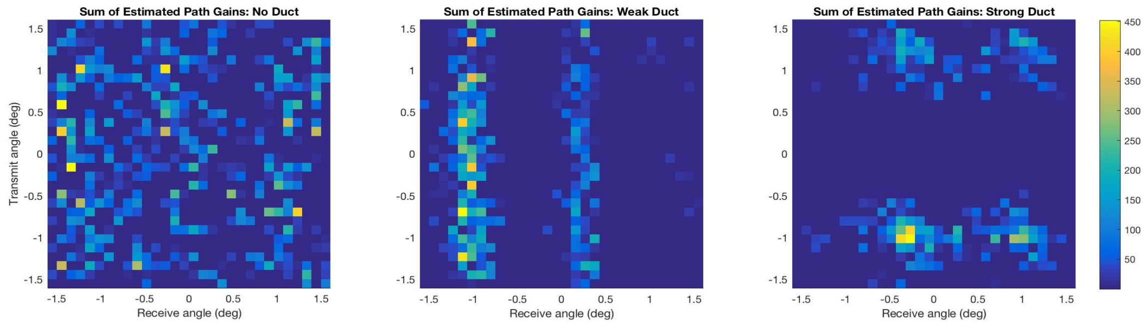

Fig. 2. Sum of estimated path gains during periods of no duct, forming duct, and strong ducting events for 2 hour windows of data.

where $||.||_2^2$ is the squared ℓ_2 norm, $||.||_1$ is the ℓ_1 norm, and μ is a positive regularization parameter satisfying $0 \leq \mu \leq 2||\mathbf{A}^H\bar{\mathbf{y}}||_\infty$ [2]. μ was set to half its maximum value.

III. EXPERIMENTAL SETUP AND DATA

The coast of southern California is known for atmospheric ducts [3], which act as leaky waveguides for EM waves. The goal of the experiment was the measuring of duct strength and frequency over the ocean. It is expected there will be high propagation loss unless a duct is present because the horizon obstructs the line of sight path.

A. Transmitters and Receivers

Data was collected from ULAs of 4 transmitters and 4 receivers. The transmitted signals were length 8192 Zadoff-Chu signals [4] sent on carrier frequency 1.385 GHz and sampled at 1.25 MSps over 40.72 km. The line of sight path between transmitter and receiver were obstructed by the horizon. Each of the receivers recorded four snapshots taken 2 s apart every fifteen minutes. Due to array spacing the maximum resolvable angles were $\phi_{max} = 1.63°$ and $\theta_{max} = 1.55°$, which is within the normal range of expected AoA and AoD's [6].

IV. RESULTS

A. Simulation

A parabolic equation (PE) algorithm [7] was run simulating the propagation loss (PL) experienced by the MIMO testbed transmitting through evaporation ducts of variable height. The PE simulation suggested PL at the receiver should vary by 17 dB depending on evaporation duct height, which agrees strongly with the experimental data.

A simulation was constructed to test the effectiveness of path recovery using (6) for signals with varying SNR and number of paths using $Q_T = Q_R = 30, L = 50$ and all path gains equal to unity. Gaussian noise was added to simulated signals to fulfill a specified SNR at the output. It was found that two variables influenced optimization accuracy of eq (6): signal SNR and number of paths. Accuracy was defined as the ℓ_2 error between $\mathbf{A}\bar{\mathbf{h}}$ and $\bar{\mathbf{y}}$, shown in figure 1.

B. Processing Pier Data

The optimization in (6) was applied to the data taken over periods where receiver SNR was either at a minimum, increasing rapidly, or at a maximum. The minimum SNR observed was 1 dB, while the maximum observed SNR was 17 dB. Almost all data collected fell in one of the two categories, the only exception were short periods where SNR increased or decreased rapidly. The low and high SNR regimes are interpreted as the existence and non-existence of ducts. Each frame of figure 2 shows the summation of 40 snapshots of data over windows of approximately 2 hours for low SNR, rapidly increasing SNR, and high SNR data.

V. CONCLUSIONS

A sparse signal model was used to characterize OTH signals in AoA and AoD space. Simulations showed that the model could accurately identify multipath signals, but decreased in accuracy as number of paths increased and SNR decreased. The model was used to identify paths taken by signals from a MIMO antenna setup in southern California, where a multipath environment was formed along with a duct.

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