Assessment of an analytical expression for an evaporation duct refractivity profile

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***Abstract*—The prediction of radar coverage for systems operating within or near the Marine Surface Boundary Layer (MSBL) requires the knowledge of the vertical refractivity profile. This vertical profile and the corresponding evaporation duct height are generally computed by the means of semi-empirical "*bulk*" models. Nevertheless, for practical purposes, it could appear very useful to describe the refractivity profile with an analytical expression. The "classical" log-linear law that has been widely used during the past decades has recently been modified and improved. This paper discusses the ability of this new expression to match a physical "bulk" model. Our study concerns both simulated vertical profiles and some corresponding propagation results in X band.**

Keywords—radar propagtion; refractivity profile; evaporation duct; analytical expression

I. INTRODUCTION

Evaporation ducts exist most of the time above the seas and the oceans. Their heights may vary from a few meters to a few tens of meters. The corresponding vertical refractivity profiles exhibit non-standard gradients that greatly influence the radar coverages at low altitudes. Within the MSBL, these profiles are modeled by applying the Monin-Obukhov similarity theory. The resulting models are generally called "*bulk*" models. The French PIRAM model is one of them. It has been widely used and validated with various sets of experimental data. Within the MSBL, we have shown that, considering propagation applications, the vertical resolution of the profiles should not be less than 1 m (Claverie, 2015 IEEE APS and URSI, 19-24 July, Vancouver). Therefore, to minimize the amount of data needed to compute the radar coverage diagrams, it would be helpful to represent the refractivity profiles by an analytical expression containing a limited number of parameters. The main purpose of this paper is to assess the ability of a recently published analytical approximation to precisely fit the physical PIRAM profiles and to update our first and preliminary results (Claverie and Hurtaud, 2016 IEEE APS and URSI, June 26 – July 1, Puerto Rico).

II. REFRACTIVITY PROFILES IN THE SURFACE LAYER

A. Basic definitions

As the tropospheric refractive index is very close to unity, the use of the refractivity *N* is generally preferred. Refractivity is a function of the following meteorological parameters: the total atmospheric pressure *p* (in hPa), the air temperature *T* (in K) and the water vapor pressure *e* (in hPa), with:

$$N = \frac{77.6}{T}\left(p + 4810\,\frac{e}{T}\right) \tag{1}$$

Therefore, refractivity depends on the height z and the function *N(z)* is referred as the vertical refractivity profile. To visualize the presence of an eventual duct for a given refractivity profile, it is more convenient to use the modified refractivity *M*, practically defined as:

$$M(z) = N + 0.157 \cdot z \tag{2}$$

If we only consider the MSBL vertical profiles, the evaporation duct height, denoted as δ, is the solution of:

$$dM/dz = 0 \tag{3}$$

B. Analytical approximations of the M profile

For near neutral atmospheric situations, the vertical M profile is quite well approximated by a log-linear law:

$$M(z) = M(0) + G_M\left[z - (\delta + z_0)ln\left(\frac{z+z_0}{z_0}\right)\right] \qquad (4),$$

where G_M is the modified refractivity gradient that would be reached at an infinite height (with a value usually closed to the standard one of -118 M/km) and z_0 is the roughness length arbitrarily fixed at 0.00015 m.

Recently, Salamon, Hansen and Abbott (SHA) published a modified expression (Salamon et al., Elec. Let., 9th July 2015, Vol 51, N°14, pp. 1119-1121):

$$M(z) = M(0) + G_M\left[z - D_p\,\frac{\left(\frac{z+z_0}{z_0}\right)^p - 1}{p}\right] \tag{5}$$

This formulation leads to the following equation for the modified refractivity gradient:

$$\frac{dM}{dz}(z) = G_M\left[1 - D_p\,\frac{\left(\frac{z+z_0}{z_0}\right)^{p-1}}{z_0}\right] \tag{6}$$

It can easily be demonstrated that, if p=0, (5) and (4) become equivalent and also that the parameters D_p and p are linked to the duct height δ by the relation:

$$D_p = [(\delta + z_0)^{1-p}] \cdot z_0^p \tag{7}$$

III. ANALYTICAL APPROXIMATION OF THE PIRAM PROFILE

We ran our "*bulk*" model PIRAM nearly 13000 times with various input meteorological situations. More precisely, we choose our inputs as follows:

- Sea temperature ∈ [5°C ; 25°C]
- Air Sea Temperature Difference ∈ [-8°C ; +4°C]
- Relative Humidity ∈ [40 % ; 95 %]
- Wind Speed ∈ [1 m/s ; 18 m/s]
- The atmospheric pressure was fixed at 1013 hPa.
- We tried to automatically avoid the unphysical combinations of these parameters.

To approximate each PIRAM profile with the SHA expression, we imposed 2 constraints:

- The SHA profile must give the same duct height δ
- The SHA profile must give the same vertical gradient at the height z_{inf} under which the PIRAM profile becomes linear.

Consequently, we searched the "best" value of the parameter p by minimizing the Root Mean Square Error (RMSE) between the PIRAM profile and the SHA profile (it can be noticed that, with the 2 former constraints, we get a unique solution for the log-linear approximation). Moreover, we defined altitude grids depending on the duct height in order to give less importance to the heights above δ. In fact, we developed two different fitting processes: the first one by working directly with the M profile and the second one by using the *dM/dz* vertical gradient. The two methods lead to slightly different solutions and RMSE values.

For all the simulated cases, we obtained a value of p between -0.5 and +0.5 ; the negative values are associated with atmospheric unstable cases (ASTD < 0°C) and the positive ones with stable cases. The fitting process based upon the M profiles gives generally lower RMSE values than the one based upon the *dM/dz* profiles.

IV. SIMULATION RESULTS

We illustrate our fitting results for the particular case defined in Fig 1. This case belongs to the 25 % worst ones in terms of RMSE values. It appears that the two profiles based upon the SHA expression lead to slightly different values of the parameter p. Both of them are closer to the PIRAM profile than the log-linear one. Moreover, it seems that the "*M* fit" is better than the "*dM/dz* fit".

With these different profiles, we simulated the propagation of an X band signal. For a particular link geometry (transmitter and receiver just above the duct), the results are plotted on Fig 2. We can notice that the "*dM/dz* fit" leads to propagation factor values very close to the original PIRAM case. We obtained the same conclusion for other link geometries. But, for other physical inputs, it is sometimes the "*M* fit" that gives the best results. We still have to investigate this point.

Furthermore, we also found that, the parameter p can be quite well empirically linked to the duct height and the Monin-Obukhov length.

V. CONCLUSION AND FUTURE WORK

Our results demonstrate that the radio refractivity profile computed by a "bulk" model can be quite well approximated by a quite simple analytical approximation. The precision obtained is in fact compatible with the measurement uncertainty of the input meteorological parameters.

Our future work will focus on the use of a similar analytical approximation to describe the subrefraction cases and on its possible adaptation to the refractivity profiles at optical wavelengths.

Moreover, as we have many experimental data sets available (meteorological and propagation measurements), we want to use this large amount of data to definitively assess the practical interest of the SHA formulation.

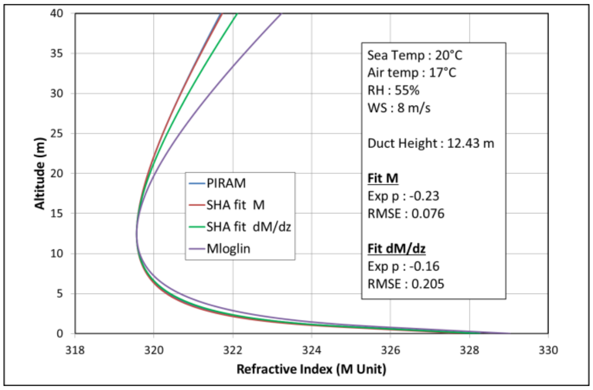

Fig. 1. Example of a PIRAM refractivity profile and the corresponding fitted profiles.

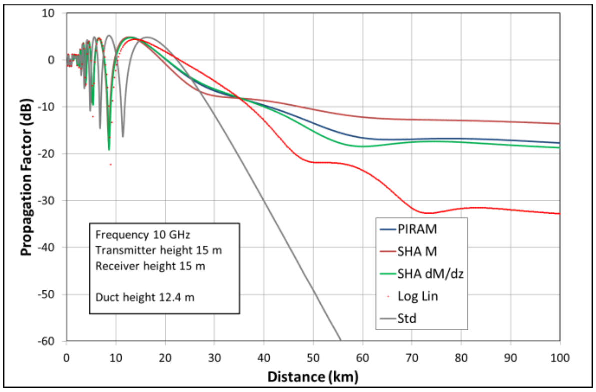

Fig. 2. Simulated propagation factors corresponding to the vertical profiles of Fig. 1 (the Std curve is the one that is obtained assuming a standard atmosphere)