# A new convenient tool for ice sheets exploration
# The fractal dimension

Gerardo Di Martino, Alessio Di Simone, Giorgio
Franceschetti, Daniele Riccio

DIETI, Università di Napoli Federico II
Naples, Italy
gerardo.dimartino@unina.it, alessio.disimone@unina.it,
gfrance@unina.it, dariccio@unina.it

Stephen D. Wall

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, USA
stephen.d.wall@jpl.nasa.gov

*Abstract*—**In this paper, we focus on the estimation of the Fractal Dimension of ice sheet surfaces from Synthetic Aperture Radar data. Thanks to the relation with the geophysical phenomena, which are responsible and drive the ice layer formation and evolution, the Fractal Dimension is a convenient parameter to classify, and discriminate icy layers of different types. Meaningful results obtained on a Sentinel-1 C-band SAR image of the Antarctica ice sheet are presented.**

*Keywords—fractal dimension, ice sheets, synthetic aperture radar*

## I. INTRODUCTION

Microwave satellite remote sensing represents a unique mean to perform all-weather monitoring of otherwise hardly reachable ice sheets and glaciers. Therefore, it is of key importance to develop sound direct models and inversion algorithms, to be used for retrieving meaningful physical parameters characterizing the observed surfaces from satellite's data. The Fractal Dimension (FD) can be considered as a concise and meaningful entity, bearing crucial information for the geometrical and, much more interesting, geophysical characterization of the surface [1]. More specifically, geologists normally use the FD to model the roughness of natural surfaces, since it is − in theory and in practical measurements − not dependent on the size of the observed surface. Therefore, it provides a powerful alternative to the classical statistical roughness descriptors (such as the height standard deviation and correlation length) that depend on both scale and size of the area on which they are estimated [2].

Regarding SAR data, several approaches have been investigated. The analysis of image texture has demonstrated to provide very useful information for the discrimination and classification of different types of icy layers [3]. In this paper, we focus on ice sheet surfaces, analyzing the potentialities of Synthetic Aperture Radar (SAR) data for the estimation of their FD. The fractal behavior of the roughness of ice sheets has been already highlighted in the literature: for instance, the fractal behavior of the arctic ice sheet has been observed on spatial scales ranging from 300 m to 10 km, by using Landsat multispectral data [4]. Seasonality, weathering, and orography are some of the main phenomena dictating essence and shape of icy layers [5]. The ice surface FD is depending on these phenomena, which contributed to its generation, and modifications: accordingly, FD is a convenient parameter for their classification and discrimination. It follows that the estimation of the FD of ice sheets, through the analysis of remotely sensed images, is a topic of great interest for glaciological applications, since in this context in situ measurements represent a very difficult task.

Recently, a technique for estimating the FD of natural surfaces from SAR data has been proposed [6]: new models for the imaging of natural surfaces have been introduced, and results regarding the FD evaluation, directly from SAR images, have been discussed. The proposed approach is applicable to local areas within the SAR image [6]: the output is the generation of a new "image", the *fractal dimension map*, i.e., a point-by-point map of the estimated FD of the imaged surface. The technique is based on the inversion of electromagnetic surface scattering models [6], so that its application to rough bare soil surfaces is straightforward.

However, in the case of ice sheets, the microwave signal can significantly penetrate the ice layer, so that the volume scattering contribution may be significant [7]. The penetration depth of the field within the ice layer is dictated by its complex dielectric constant, which depends on ice density, chemical composition, and temperature. Whenever the penetration depth is relevant, the possible presence of balls, impurities, lack of ice homogeneity, and limited thickness of the ice profile (i.e., the boundary reflection) are the dominant components of the scattered field, rather than the ice surface roughness. In this case, the backscattered signal may be dominated by the volume scattering contributions.

In the following section, we present results regarding the estimation of the FD from Sentinel-1 C-band SAR data, acquired on the Antarctica ice sheet. Additional results will be available at the Symposium venue.

## II. RESULTS

Recently, a complete direct analytical model of the SAR imaging process of natural surfaces has been developed [6]. This model provides the expression of the power spectral densities of range and azimuth cuts of a SAR image in closed form. It is demonstrated that, assuming a small-slope regime

for the surface, for sufficiently low wavenumbers the range-cut spectrum of a SAR image of a natural surface exhibits a power-law behavior. Hence, in a log-log plane, this spectrum is represented by a linear graph, whose slope is related to the FD of the observed scene. Therefore, the FD can be retrieved through a simple linear regression.

As a test case, we consider the Sentinel-1 C-band SAR image of the Antarctica ice sheet, whose intensity is shown in Fig.1. The considered image has a resolution of about 22m×5m in azimuth-ground range, and covers an area of about 230 km². Many different textural features are present on the considered image. To perform the proposed fractal analysis, we selected ten subsets of 512×512 pixels from the original image (highlighted in red boxes in the figure), which are representative of the main different characteristics present in the area. Two examples of image subsets are shown in Fig.2, along with the corresponding estimated FD maps. A statistical analysis of the obtained FD maps has been carried out: as an example, in Fig.3 the histograms relevant to the two FD maps in Fig.2 are reported. A first key result is that the values of the FD are all enclosed in the range (2-2.5), demonstrating the fractal behavior of the observed surface. Moreover, it can be observed that the average values of the estimated FD are very similar: indeed, they are equal to 2.31 for Crop_7_FD and 2.30 for Crop_9_FD. Similar results are obtained for the other image subsets.

Additional results, to be presented at the Symposium, will involve the estimation of the FD map using a sliding window of smaller size (we used here a window of size 51×51 pixels), to provide a better analysis of fine grain features (e.g. see Crop_7 in Fig.2). The effects of volumetric scattering on the proposed procedure will be illustrated, too.

## III. CONCLUSION

As a summary, this contribution highlights the relevance of this novel parameter FD for detection and analysis of ice sheets' discrimination and classification. Antarctica data are processed, generating a number of results, convincing about validity and convenience of the proposed framework.

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Crop_7     Crop_7_FD

Crop_9     Crop_9_FD

Fig. 2. Two examples of cropped subsets: the numbers refer to the red boxes in the previous figure. The cropped SAR images are reported on the left, and the correesponding estimated FD maps on the right.
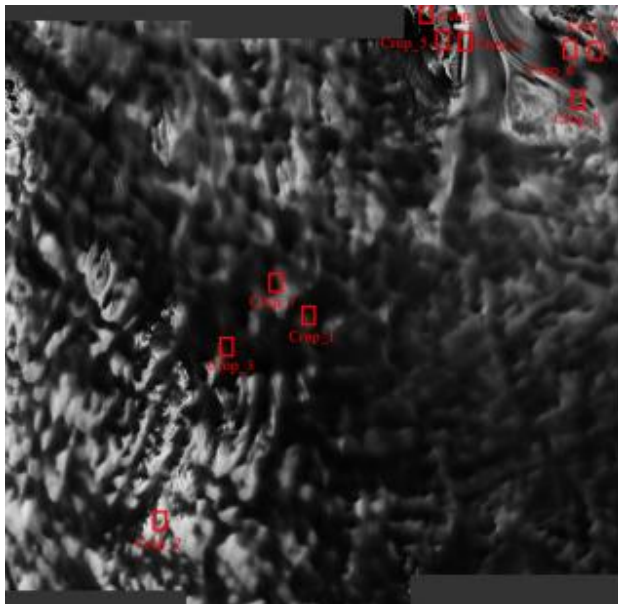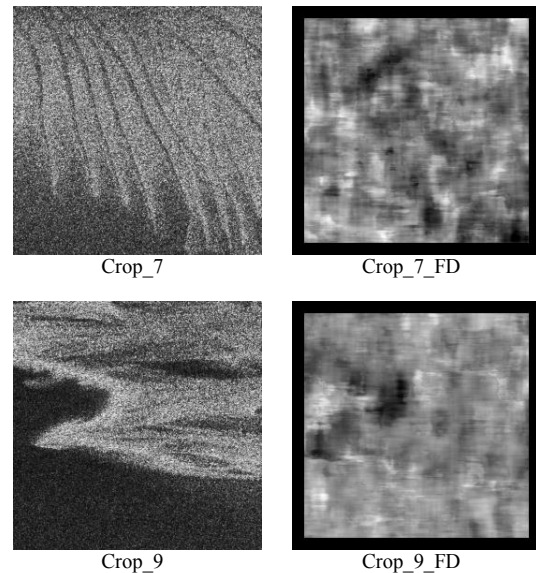


Fig. 1. Sentinel-1 image considered for the study. The considered regions are highlighted with red boxes. A multilook of 1×4 (azimuth-range) has been applied, for visualization purposes, to obtain a ground range square pixel. Range is the horizontal, and azimuth the vertical coordinate. Near range is on the left.
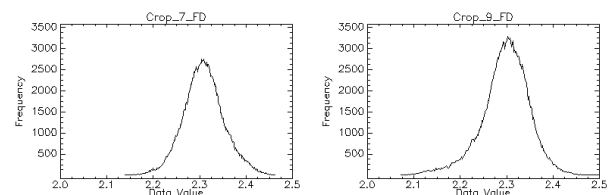


Fig. 3. Histograms of the FD maps shown in the previous figure.