Supershaped Complementary Split-Ring Resonators

E. Rubio Martínez-Dueñas
Electromagnetic Department
University of Granada
Granada, Spain

C. Moreno de Jong van Coevorden and D. Caratelli.
The Antenna Company
Eindhoven, The Netherlands
carlos.vancoevorden@antennacompany.com

***Abstract*—This paper investigates the possibility of using complementary split ring resonators based on Gielis transformation as basic elements for the design of microwave filters implemented in planar technology. From the electromagnetic simulation of these structures, suitable equivalent circuit models are extracted and analyzed. Physical prototypes are fabricated and tested for design validation. The study confirms that the adoption of supershaped geometries enables the synthesis of very compact microwave filters.**

Keywords—Gielis formula, microwave filters, CSRR.

I. INTRODUCTION

Split ring resonators (SRRs) have been originally proposed as metamaterial structures featuring negative permeability. Complementary split-ring resonators (CSRRs), which are manufactured by etching the complementary version of SRRs on printed circuit boards (PCBs), are typically used as negative-permittivity components in planar left-handed structures [1].

The use of SRRs and CSRRs as the basic resonant unit of planar microwave filters has raised a growing interest due to their smaller size when compared to conventional resonators, this allowing for the design of compact structures with high performance and controllable characteristics [1].

Canonical circular or square geometries are typically adopted for the design of the considered class of resonators (see Fig. 1), whereas fractal geometries have been recently proposed for the realization of miniaturized CSRRs [2].

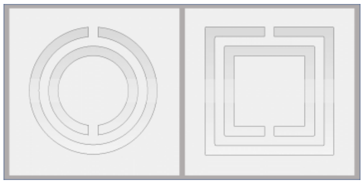

Fig. 1. Canonical circular (left) and square (right) CSRR.

Square and circle may be regarded as special cases of the Gielis transformation [3], so the question arises whether higher-order supershaped geometries could provide benefits in the design of compact planar microwave filters. The goal of this study is to analyze the performance of band-reject CSRR filters whose geometry is based on Gielis transformation.

The paper is organized as follows. A brief description of the geometry of the considered CSRR unit cells, as well as of the equivalent circuit models used to investigate the relevant transmission properties is given in Section II. Finally, results are presented and discussed in Section III.

II. SUPERSHAPED CSRRS

A. Structure Geometry

Gielis transformation is a generalization of Lamé curves that can be used to describe a wide variety of complex shapes found in Nature. Its general expression is reported in [3]. In this study, we are making use of the following particular case:

$$r(\theta,n,m) = R\left[\left|\cos(m\theta/4)\right|^{n} + \left|\sin(m\theta/4)\right|^{n}\right]^{-1/n} \tag{1}$$

r and θ denoting polar coordinates. In (1) the parameter n is assumed to be equal to 128, whereas m is a positive integer number, and R is a suitable scaling factor. Various closed curves can be generated by choosing different values of the design parameter *m* (see Fig. 2). It can be easily found out that the supershaped curves described by (1) feature length that increases with *m* while, for *m>0,* the area is constant.

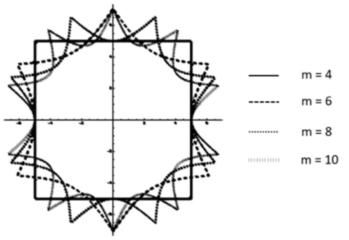

Fig. 2. Supershaped curves described by (1) for different values of *m*.

B. Equivalent Circuits

The microwave structures investigated in this study consist of a microstrip transmission line loaded with a supershaped CSRR described by (1) etched in the ground plane. An example of such structure for *m*=6 is given in Fig. 3.

Thanks to the small electrical size of CSRRs at resonance ($\sim\lambda/10$), the considered structures can be conveniently described by means of suitable lumped-element equivalent circuits. Several equivalent circuit models have been proposed in the scientific literature. Baena et al. [1] modeled the behavior of the structure by using the circuit shown in Fig. 4 (left), where L_c and C_c denote the equivalent inductance and capacitance of the CSRR setting the stop-band frequency, L_c being related mainly to the length of the CSSR. The capacitance C, which is related to the area of the CSRR, models the coupling between the CSRR and the transmission line which is characterized by inductance L. Li et al. [4] proposed the equivalent circuit model shown in Fig. 4 (right), consisting of a series LC resonator (L_1 and C_1) with a capacitance C_2 connected in parallel, as well as

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50Ω transmission lines having length *d*. A detailed description of the parameter extraction process can be found in [1], [4].

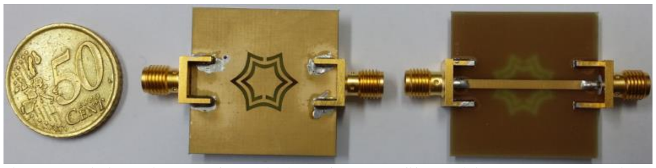

Fig. 3. Fabricated super-formula-based CSRR filter with *m*=6. Bottom view (center). Top view (right).

III. RESULTS

In order to investigate the resonant properties of Gielis CSRRs, several designs have been made and fabricated with the structure parameter *m* ranging from 5 to 10. Two concentric supershaped rings, separated by 0.75mm, are etched on the ground plane of the printed circuit. Each ring is characterized by width of 0.75mm and a gap of 0.75mm. In all the designs, a dielectric laminate having relative permittivity of 3.78, and loss tangent tanδ=0.025 has been adopted. Printed on circuit substrate is a microstrip line featuring a width of 1.835mm in such a way as to achieve a nearly 50Ω characteristic impedance. The thickness of the metal layer is 35μm. SMA connectors are soldered at both ports. The structures have been numerically simulated using the commercially available electromagnetic solver *CST Microwave Studio*, whereas the scattering parameters of the fabricated prototypes have been measured by means of a vector network analyzer (VNA).

Following the procedure described in [1] and [4], equivalent circuit models of the proposed supershaped CSRR-loaded circuits have been extracted from the collected numerical data. The circuit parameters are given in Table I for each design. It is worth noting that the value of L_c increases with the length of the CSRR, while the capacitance C_c slowly varies around a mean value of about 0.275pF.

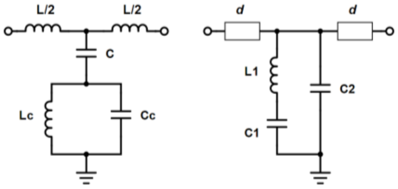

Fig. 4. Equivalent circuit models of CSRR. Baena et al. (left). Li et al. (right).

Fig. 5 shows the simulated and measured insertion loss ($|S_{21}|$) versus frequency of designed two-port networks. As expected, the resonance frequency of the considered class of CSRRs decreases with *m*. A good agreement between numerical and experimental results can be noticed.

As an example, Fig. 6 shows the frequency-domain behavior of the magnitude of the coupling coefficient $|S_{21}|$ and input reflection coefficient $|S_{11}|$ of the supershaped CSRR with *m=10*, as simulated with *CST Microwave Studio*, measured, as well as evaluated with both Li's and Baena's equivalent circuits. The agreement between numerical and experimental results is pretty good, minor discrepancies in return loss being attributed to tolerances in the fabrication process and parasitic effects associated with the connector-line transition. As for the equivalent circuits, both proposed models can predict the stop-band response with a reasonably accuracy, though Li's equivalent circuit shows a better agreement with simulated data across a broader frequency band. On the other hand, Baena's circuit provides a useful insight in terms of the equivalent inductance and capacitance, L_c and C_c respectively, of the CSRR.

TABLE I. Equivalent circuit parameters of supershaped CSRRs as a function of the geometrical parameter *m* (C_s in pF, L_s in nH)

m	L	C	Lc	Cc	L1	C1	C2
5	3.85	0.95	5.59	0.27	8.31	0.83	-2.51
6	3.54	1.06	5.62	0.22	7.98	0.90	-2.51
7	4.18	0.91	6.38	0.28	9.44	0.80	-2.51
8	4.33	0.87	6.70	0.30	10.28	0.76	-2.53
9	4.45	0.89	6.98	0.31	10.35	0.81	-2.63
10	4.37	0.86	7.92	0.27	11.58	0.78	-2.59

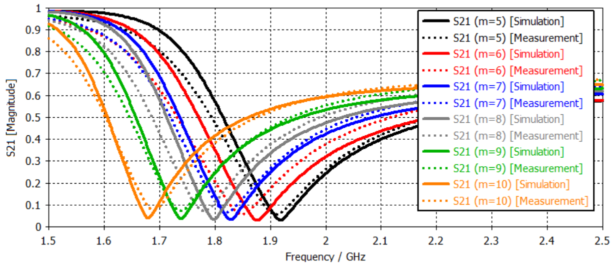

Fig. 5. Transmission coefficient of CSRRs with *m*∈[5,10]. Simulated results (solid lines). Measurements (dotted lines).

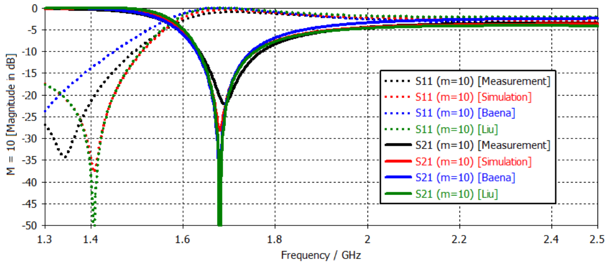

Fig. 6. $|S_{21}|$ (solid line) and $|S_{11}|$ (dotted line) of the Gielis CSRR with *m*=10.

In order to achieve enhanced stop-band filtering characteristics, novel designs based on integration of periodic patterns of supershaped CSRRs are being investigated. More details about this specific research study will be given during the conference.

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