Adaptive Generation of Excitation-Independent Characteristic Basis Functions

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***Abstract*—The Characteristic Basis Function Method (CBFM) has been widely used in recent years to solve a variety of electromagnetic scattering problems because it offers a way to direct-solve large problems without the use of iteration, enabling it to not only handle multiple excitation (r.h.s.) problems in an efficient manner, but to also bypass the issues of ill-conditioning as well as generation of preconditioners. In the past, the choice of number of plane wave excitations used to generate the excitation-independent CBFs have been made empirically, on an *ad hoc* basis, rather than systematically. This paper presents an adaptive approach for generating the CBFs, which provides a systematic way to determine of the number of plane waves needed for CBF generation, and this in turn renders the CBFM computationally efficient.**

Keywords—Method of Moments (MoM); Characteristic Basis Function Method (CBFM); Fast Adaptive Cross Approximation (FACA)

I. Introduction

The Characteristic Basis Function Method (CBFM) [1] has been widely used in recent years to solve a variety of electromagnetic scattering problems. The CBFM is a domain decomposition algorithm, which utilizes a set of excitation-independent macro basis functions, referred to as Characteristic Basis Function (CBFs), to generate a reduced matrix whose rank is much smaller than that of the matrix for the same problem generated by using low-level sub-sectional basis functions, such as the Rao-Wilton-Glisson (RWG) [2] functions. Several efficient methods, e.g., [1], [3]-[9], have been proposed in recent years to generate the excitation-independent CBFs efficiently for each block in which the original geometry has been decomposed. However, in all of these methods, the number of plane waves employed to generate the CBFs are usually chosen empirically, since it is difficult for the users to estimate this number *a priori*. To resolve this issue, a systematic approach is proposed in this work to generate the CBFs by setting the number of plane waves adaptively.

II. Adaptive Generation of CBFs

Let us consider, as an example, the *i*th block which is extended by a fixed border region in all directions to mitigate the problem of fictitious singularities which appear in the CBFM because of the truncation of the original geometry into blocks. The proposed adaptive method is based on an iterative approach, which comprises three steps at the *n*th iteration (*n*=1, 2, 3, …) stage, as described below.

Step 1: Select $N_{PW}^{(n)}$ plane waves from the plane wave spectrum (PWS).

$$N_{PW}^{(n)} = 2^{n-1} \times N_{PW}^{(1)}, \tag{1}$$

where $N_{PW}^{(1)}$ is a relatively small number.

Step 2: Carry out an ACA decomposition [11]-[13] of the extended excitation matrix $\mathbf{E}^{(n)}$, which is obtained by employing plane waves selected in Step-1 to illuminate the extended *i*th block.

$$\mathbf{E}^{(n)} \approx \mathbf{U}^{(n)}\mathbf{V}^{(n)}, \tag{2}$$

where $\mathbf{U}^{(n)}$ and $\mathbf{V}^{(n)}$ are the ACA decomposition matrices of $\mathbf{E}^{(n)}$, and $S_{PW}^{(n)}$ denotes the effective rank of $\mathbf{E}^{(n)}$ evaluated by the ACA.

Step 3: Terminate the iteration process by using the criterion:

$$\left|S_{PW}^{(n)} - S_{PW}^{(n-1)}\right| \Big/ \left|N_{PW}^{(n)} - N_{PW}^{(n-1)}\right| \le \beta, \tag{3}$$

where $S_{PW}^{(0)} = 0$, $N_{PW}^{(0)} = 0$, and β is a small constant, which is typically set to 0.1 following the recommendation given in [10]. Once the criterion in (3) is satisfied, we can argue that the number $S_{PW}^{(n)}$ has become stable, and would change little if we were to further increase $N_{PW}^{(n)}$. This is because $\left|S_{PW}^{(n)} - S_{PW}^{(n-1)}\right| \Big/ \left|N_{PW}^{(n)} - N_{PW}^{(n-1)}\right|$ approximately denotes the derivative of the function $S_{PW}^{(n)} = f\left(N_{PW}^{(n)}\right)$; hence, we can argue that the selected number of $N_{PW}^{(n)}$ plane waves are adequate for generating the CBFs.

We assume that the adaptive method exits following the n_tth iteration step. The selected $N_{PW}^{(n_t)}$ plane waves are then used to generate the initial CBFs, and the method given in [9] is employed to generate the remaining retained CBFs.

To determine the number of plane waves efficiently, and to reduce the size of $\mathbf{E}^{(n)}$ in the adaptive method, the basis functions can also be sampled in the same manner as is done in the fast adaptive cross approximation (FACA) [10][13].

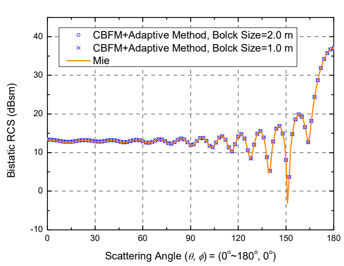
III. Numerical Results

To test the validity of the proposed adaptive method, the bistatic radar cross sections (RCSs) of a PEC sphere with the radius of 2.5 λ is calculated, which is discretized into 28890 RWGs. The sphere is illuminated by an $\hat{\mathbf{x}}$ -polarized plane wave with the incident angle $(\theta,\phi)=(0^\circ,0^\circ)$. The threshold τ of the truncated SVD for generating the CBFs is set to 10^{-3}. The threshold of the ACA in the adaptive method is set to 0.1τ .

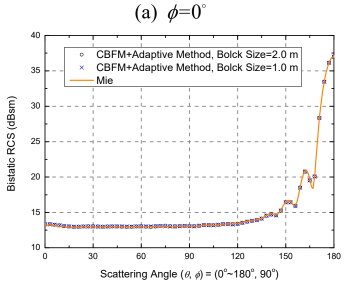
The dimensions of the blocks used to generate the CBFs is set to two values, namely 1.0m and 2.0m. In the proposed method, it is not necessary to set the number of plane waves for generating the initial CBFs for the two cases, since we can determine this number in an adaptive manner. Table 1 summarizes the numbers of blocks, the numbers of CBFs, and the error levels of RCSs at ϕ=0° when compared with the Mie solution for different block sizes. As shown in Fig. 1 and Table I, the RCS values obtained by using the CBFM, combined with the adaptive method for different block sizes, agree well with the Mie series solutions.

TABLE I. The numbers of blocks, the numbers of CBFs and the erroes of RCSs for different block sizes

Block Size	No. of Blocks	No. of CBFs	Error in RCS
1.0 m	128	5767	0.08 dBsm
2.0 m	26	3053	0.07 dBsm



(a) ϕ=0°



(b) ϕ=90°

Fig. 1. Bistatic RCSs of a PEC sphere with radius 2.5 λ . (a) ϕ=0° ; (b) ϕ=90° .

IV. Conclusion

In this paper, an adaptive method has been proposed for generating excitation-independent CBFs, in which the plane wave spectrum for generating the initial CBFs is adaptively sampled, obviating the need to set the number of plane waves empirically. Numerical results validating that the accuracy of the CBFM based on this approach have been included.

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