Synthesis of 3D-Printed Dielectric Lens Antennas Via Optimization of Geometrical Optics Ray Tracing

Jordan Budhu and Yahya Rahmat-Samii
Electrical Engineering
UCLA
Los Angeles, California, USA
jordan.budhu@ucla.edu, rahmat@ee.ucla.edu

***Abstract*—A numerical recipe is presented to synthesize symmetric lenses via a hybridization of two numerical methods for potential space borne applications requiring a conical pattern scan. The algorithm is based on the concepts of Geometrical Optics Ray Tracing and Particle Swarm Optimization of a 10-term Quadratic-Tapered Legendre Series Expansion of the lens surface topology parameterized as a Body-of-Revolution to obtain azimuthal symmetry. Full-Wave analysis is used to validate the technique. An example of an on-axis fed shaped lens optimized for uniform phase is presented.**

Keywords—Geometrical Optics; Ray Tracing; Particle Swarm Optimization; Lens Antenna; Shaping

I. Introduction

In the past ray tracing techniques have been successful in the synthesis of reflector antennas which yield high aperture efficiencies [1]. In addition, with the advent of 3D printing technologies the fabrication of shaped dielectric lens antennas has become easier than ever before. In light of these two observations, a new algorithm to synthesize shaped dielectric lens antennas is warranted. In this paper, a novel algorithm hybridizing Geometrical Optics ray tracing techniques and Particle Swarm Optimization is presented. The algorithm allows one to achieve high efficiency lens antennas, an example of a symmetric lens is presented and results compared with Full-Wave simulation tools.

II. Lens Synthesis, Analysis, and Design Procedure

The hybrid design scheme and numerical algorithm adopted is shown in Fig. 1.

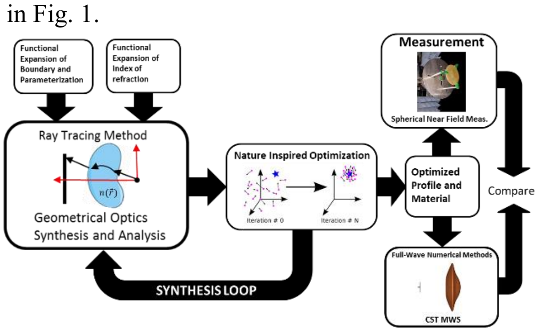

Fig. 1. Lens Design Procedure Flowchart Showing the Synthesis Loop Between Geometrical Optics and Particle Swarm Optimization. The Final Design is Validated Via Measurements and Full-Wave Simulations.

A. Functional Expansion and Parameterization of Lens

In order for the lens to obtain a unique shape for both the upper and lower hemispheres, a different expansion is performed for each hemisphere. The expansion takes the following form

$$f_{\pm}(\rho)=\pm\left(a_0\sqrt{1-\left(\frac{\rho}{a}\right)^2}+\sum_{n=1}^{9}a_nP_{2n}\left(\frac{\rho}{a}\right)\left(1-\left(\frac{\rho}{a}\right)^p\right)^{1/p}\right) \tag{1}$$

where $P_{2n}(x)$ are the Legendre Functions, and thus the expansions above can be seen as a quadratic-tapered Legendre-Series Expansion. The expansion functions are chosen to be even ordered functions only as to force a zero tangent as $\rho\to 0$ such that the body of revolution is smooth about the revolution axis. The taper is added for two reasons. First in order to force the expansion functions to go to zero at the boundary of the lens (normally the Legendre Functions attain a value of +1 at the ends of their domain of definition) and second to force an infinite tangent as $\rho\to a$ at the rim of the lens such that the upper and lower surfaces join smoothly. A 10-term expansion was found to be suitable for this application. Once the generating curve is produced via (1) and (2), the upper and lower surfaces are parameterized as a BOR using

$$\vec{r}(\rho,\phi)=\left[\rho\cos(\phi),\rho\sin(\phi),z_c\pm f_{\pm}(\rho)\right] \tag{2}$$

and joined to create the surface profile of the shaped lens.

B. Ray Tracing

Launched rays are traced to the aperture using the concepts of Geometrical Optics [2]. Fig.2 shows schematically the concepts involved.

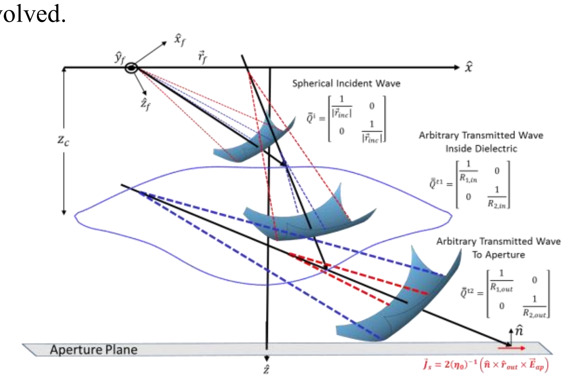

Fig. 2. Geometrical Optics Ray Tracing Diagram. Rays are Traced to the Aperture and Characterized by the Wavefront Curvature Matrix. Equivalent Currents are Generated in the Aperture Plane of which the Far Field is Calculated From.

A wavefront locally and differentially around an axial ray is described by a Curvature Matrix Q. The wavefront and/or surface is approximated as a quadratic form and thus contains 2 degrees of freedom, the radii of curvatures. As the wavefront spreads throughout space, these radii of curvatures become larger and the power density decreases proportionately. A divergence factor can then be defined as

$$DF = \sqrt{\frac{\det \overline{\overline{Q}}(2)}{\det \overline{\overline{Q}}(1)}} \quad (3)$$

which is seen as a conservation of energy statement for the energy in a flux tube of rays. With the definition of (3), one can determine the electric field at point 2 with knowledge of the field at point 1 via

$$\vec{E}(\vec{r}_2) = \vec{E}(\vec{r}_1) \cdot DF \cdot e^{-jk\left|\vec{r}_2 - \vec{r}_1\right|} \quad (4)$$

Note, the polarization state is maintained along the ray. The transmitted curvature matrices are found by phase matching the incident wavefront tp the transmitted wavefront at all points on the surface of the scatterer, in this case the surface of the lens, and is given explicitly by

$$\overline{\overline{Q}}^t(P) = \frac{k_0\left(\left(\overline{\overline{P}}^t\right)^{-1}\right)^T\left(\overline{\overline{P}}^i\right)^T\overline{\overline{Q}}^i(P)\,\overline{\overline{P}}^i\left(\overline{\overline{P}}^t\right)^{-1} + \left(k_0 p_{33}^i - k_d p_{33}^t\right)\left(\left(\overline{\overline{P}}^t\right)^{-1}\right)^T\overline{\overline{Q}}^{\Sigma}\left(\overline{\overline{P}}^t\right)^{-1}}{k_d} \quad (5)$$

Note, the superscripts i indicate 'incident', Σ indicates surface, and t indicates transmitted. Also, in (5) the $\overline{\overline{P}}$ matrices are projections of the quadratic form coordinate axes of the wavefronts onto those of the surface. Rays are traced to the aperture plane and equivalent currents are formed. The far-field pattern is then computed via the Fourier Transformation of the aperture field magnitude and phase.

C. Particle Swarm Optimization

Optimization is performed via the Particle Swarm Optimization Algorithm in order to obtain the best geometry[3]. The linking of Geometrical Optics Ray Tracing and that of the Particle Swarm Optimization Routine is shown schematically in Fig. 3.

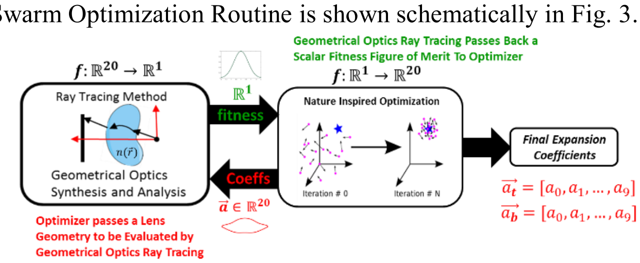

Fig. 3. Block Diagram Showing Linkage Between Geometrical Optics and Particle Swarm Optimization

The optimizer will intelligently search the whole of the solution space to find the optimum set of expansion coefficients for a given cost function. The cost function used to obtain the lens geometry is shown below

$$fitness = PhaseStdDev + 3*(1 - RayRatio) + 0.2*VolumeRatio \quad (6)$$

The First Term in (6) is the standard deviation of the aperture field phase, the second term tries to minimize the number of totally internally reflected rays, and the last aims to reduce the overall lens weight.

III. Results

As an example, a shaped on-axis fed dielectric lens antenna was designed using the GO/PSO hybrid numerical algorithm of section II. The resultant lens was fabricated using 3D printing technologies. The results of the optimization are shown in Fig. 4. The feed was linearly polarized rectangular microstrip patch located z_c = 8cm away from the geometric center of the lens and made to resonate at 13.4GHz. The lens diameter is 12cm. The optimizer converged after only 1000 iterations.

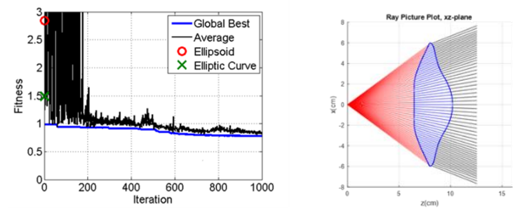

Fig. 4. Particle Swarm Optimization Convergence Plot and Final Geometry Optimized Profile Ray Picture Plot.

The optimized lens was then simulated with CST MWS and results compared for that of the Geometrical Optics Ray Tracing. The results are given Fig. 5.

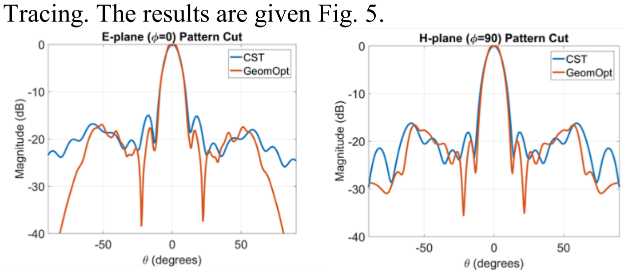

Fig. 5. CST MWS vs. Geometrical Optics Ray Tracing Dielectric Lens Antenna Far Field Radiation Patterns.

The peak directivity of the shaped lens is 12.5 dB higher than the feed patch. The synthesis of shaped dielectric lenses by ray tracing is shown to be accurate as the results of CST MWS and the ray tracing algorithm agree quite reasonably.

Acknowledgment

This work was supported in part by a grant under contract with the Jet Propulsion Laboratory. The authors would like to acknowledge fruitful discussions with colleagues at Jet Propulsion Laboratory.

References

[1] P.S. Kildal, "Synthesis of Multireflector Antennas by Kinematic and Dynamic Ray Tracing, IEEE Trans. Antennas and Propagation, Vol. 38, No. 10, pp. 1587–1599, Oct. 1990.

[2] G. Deschamps, "Ray Tecniques in Electromagnetics", Proc. of the IEEE, Vol. 60, No. 9, pp. 1023–1035, Sep. 1972.

[3] J. Robinson and Y. Rahmat-Samii, "Particle Swarm Optimization in Electromagnetics," IEEE Trans. Antennas and Propagation, Vol. 52, No. 2, pp. 397–407, Feb. 2004.