Power & Energy Relations for Macroscopic Dipolar Continua

Arthur D. Yaghjian
EM Research Consultant, Concord, MA, USA, a.yaghjian@comcast.net

Maxwell's equations in macroscopic dipolar continua can be used to derive Poynting's theorem as

$$-\int_S \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)] dS = \int_V \left[\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \cdot \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \cdot \mathbf{H}(\mathbf{r},t) \right] dV \quad (1)$$

with

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}\,, \quad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \quad (2)$$

where ϵ_0 and μ_0 are the free-space permittivity and permeability. The unit normal $\hat{\mathbf{n}}$ to the closed surface S points out of its volume V so that the left-hand side (and thus the right-hand side) of (1) is equal to the instantaneous electromagnetic power flow entering the volume V. Integrating (1) from time t_0 when the macroscopic fields are zero to the present time t gives an electromagnetic energy density $W(\mathbf{r},t)$ on the right-hand side of (1) equal to

$$W(\mathbf{r},t) = \int_{t_0}^{t} \left[\frac{\partial \mathbf{D}(\mathbf{r},t')}{\partial t} \cdot \mathbf{E}(\mathbf{r},t') + \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r},t) \right] dV dt'\,. \quad (3)$$

If it is assumed that that the electromagnetic energy in the polarized continuum is at least as great as that in a vacuum, and if $\mathbf{E}$ and $\mathbf{H}$ are taken as the primary fields (since $\mathbf{E} \times \mathbf{H}$ is the electromagnetic power flow), then (3) yields the inequality

$$\int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t} \cdot \mathbf{E}(\mathbf{r},t') + \mu_0 \frac{\partial \mathbf{M}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r},t) \right] dV dt' \geq 0\,. \quad (4)$$

If, however, one takes the view that all known magnetization is produced by Amperian magnetic dipoles (circulating electric current) and thus $\mathbf{E}$ and $\mathbf{B}$ are the primary fields, then (3) yields instead of (4)

$$\int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t} \cdot \mathbf{E}(\mathbf{r},t') - \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{M}(\mathbf{r},t) \right] dV dt' \geq 0\,. \quad (5)$$

Neither one of the inequalities in (4) or (5) are universally valid because the simple hypothetical example of a causal material with constitutive relations $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$ with constant permittivity ϵ and constant permeability μ reveals from (4) that

$$\epsilon \geq \epsilon_0\,, \quad \mu \geq \mu_0 \quad (6)$$

and from (5) that

$$\epsilon \geq \epsilon_0\,, \quad \mu \leq \mu_0\,. \quad (7)$$

This would suggest that (4) and (5) may be valid for magnetic and diamagnetic materials, respectively. In the talk, sufficient conditions will be given for (4) and (5) to be derived from the microscopic Maxwell equations obeyed by the electric and magnetic dipoles that comprise the macroscopic dipolar continuum.