Numerical Computation of Near-Singular and Near-Hypersingular Integrals in Higher Order Method of Moments Using Curved Quadrilateral Patches

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In numerical techniques based on the method of moments (MoM) in the surface integral equation (SIE) formulation in the frequency domain, special attention must be paid to achieving high accuracy, which includes advanced methods for numerical computation of singular and near-singular integrals defined on MoM-SIE patches. The techniques for dealing with such integrals, which arise for zero or small source-to-field distances in computing the MoM matrix entries, can broadly be classified into singularity extraction or subtraction methods and singularity cancellation or coordinate transformation methods. Also, when a MoM-SIE method is aimed at analysis of both metallic and dielectric/magnetic structures, such generality in electromagnetic MoM-SIE simulations increases the singularity of the integral kernel, and requires special treatment of highly singular integrals. Finally, this problem is even more pronounced when higher order basis functions are used for the approximation of electric and magnetic equivalent surface currents in the MoM-SIE method and when such functions are defined on curved patches.

This paper presents a novel method for numerical computation of near-singular (potential) and near-hypersingular (field) integrals defined on Lagrange-type generalized curved parametric quadrilateral MoM-SIE surface elements of arbitrary geometrical orders with polynomial basis functions of arbitrary current-approximation orders. The integrals are evaluated using a method based on the singularity extraction, which consists of analytical integration of a principal singular part of the integrand over a (generally not rectangular) parallelogram whose surface is close to the surface of the generalized quadrilateral near the singular point and numerical integration of the rest using Gauss-Legendre quadrature formulas.

The majority of the existing extraction techniques, used in MoM-SIE modeling so far, have been developed for planar triangular patches involving low-order basis functions. Few of those have been extended to curved patches but without really taking into account the curvature of the surface. The presented integration technique considers the curvature of the patch by extracting multiple terms in the evaluation of the principal singular part. Further, the theory behind the extraction technique has been extended to consider integrals with higher order basis functions.

Numerical examples demonstrate fast convergence of the novel integration method with increasing the orders of Gauss-Legendre integration formulas, i.e., numbers of integration points, over quadrilateral patches, in a variety of cases. Integrals are calculated over curved patches with curvature ranging from flat or almost flat patches to those with very pronounced curvature, such as spherical patches. Examples show steady behavior of the integration method for arbitrary choice of the location as well as the distance of the near-singular point. Further, examples investigate the numerical behavior of integrals with different choices of basis functions varied from constant approximations to very-high-order polynomial bases, and for elements with sizes varied from electrically very small to electrically large quadrilaterals extending to a couple of wavelengths in each dimension.